Sumcheck Arguments and Lattice-based Succinct arguments

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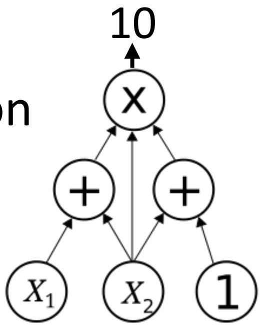
Katerina Sotiraki (Yale University)

https://ia.cr/2021/333
https://ia.cr/2023/930

Slides by Jonathan Bootle

Succinct arguments

Common input



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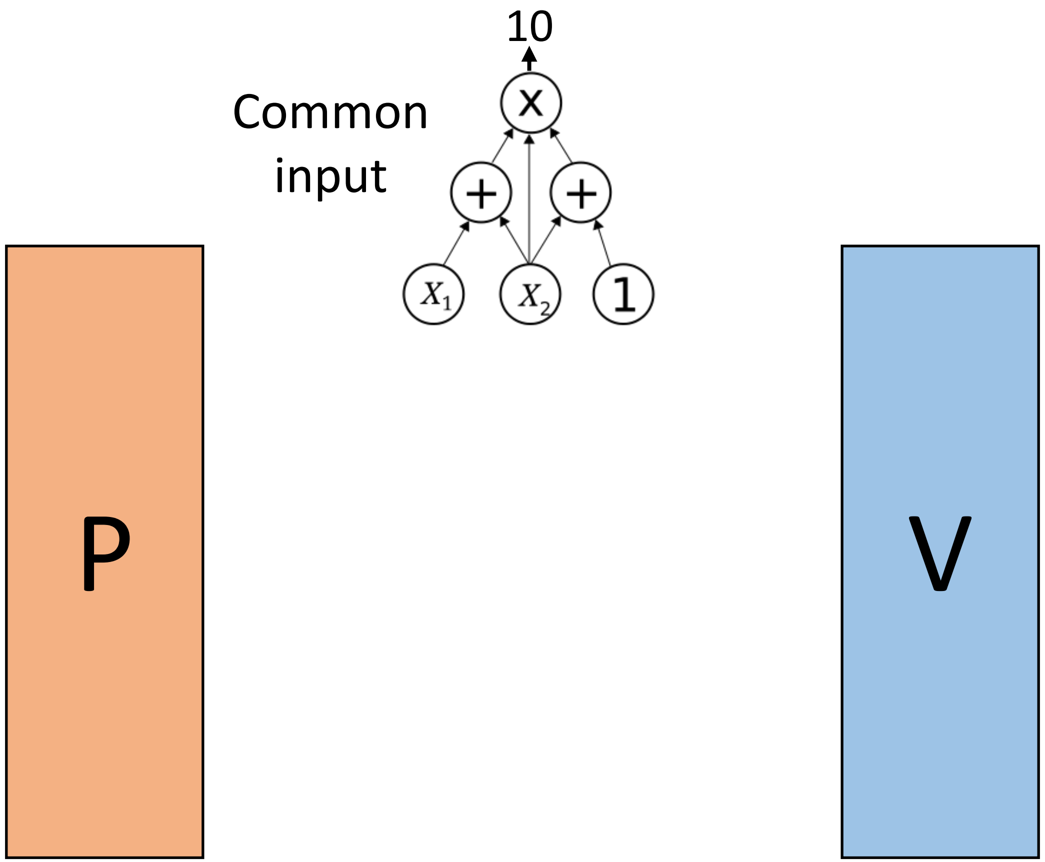

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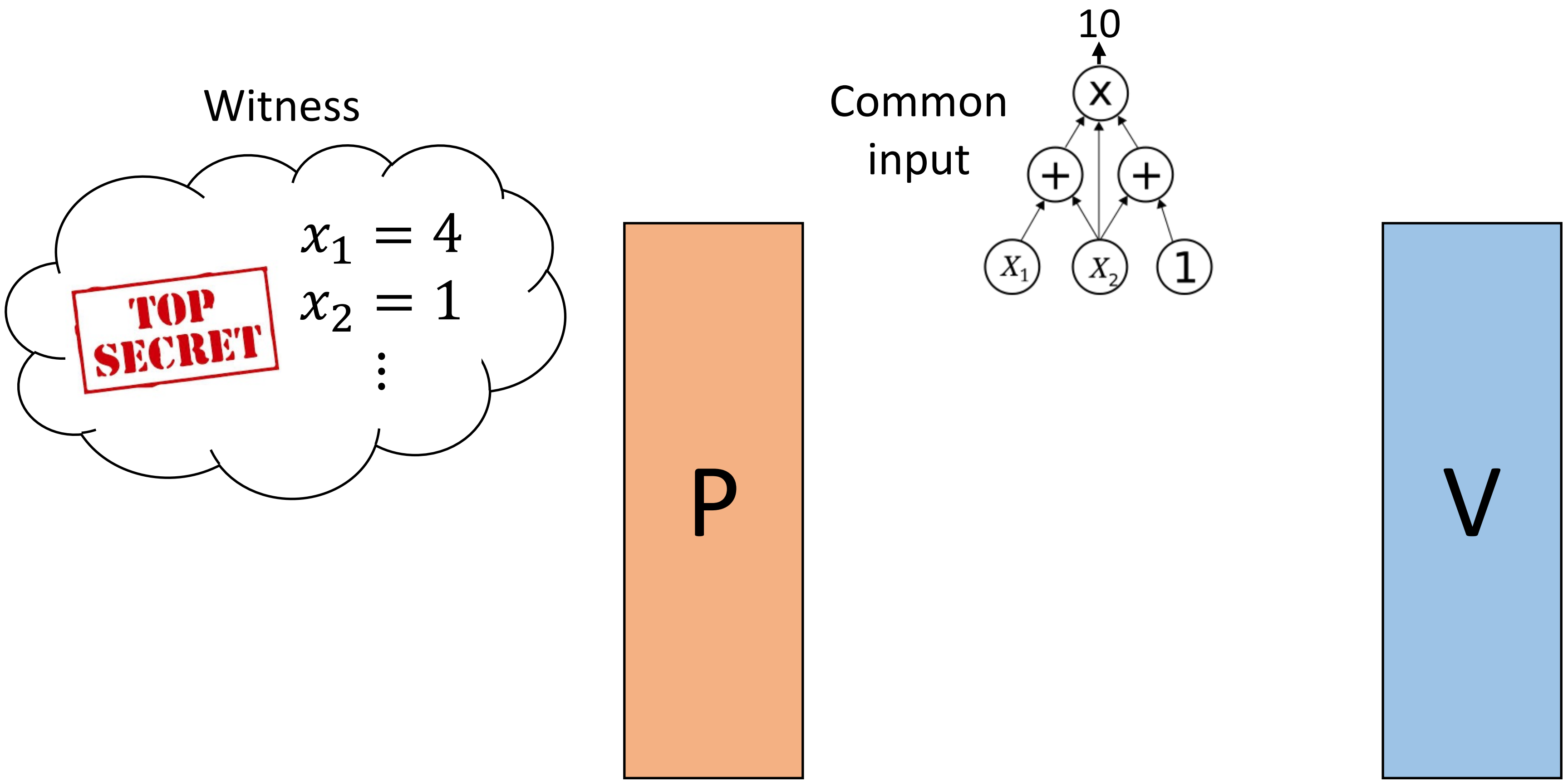

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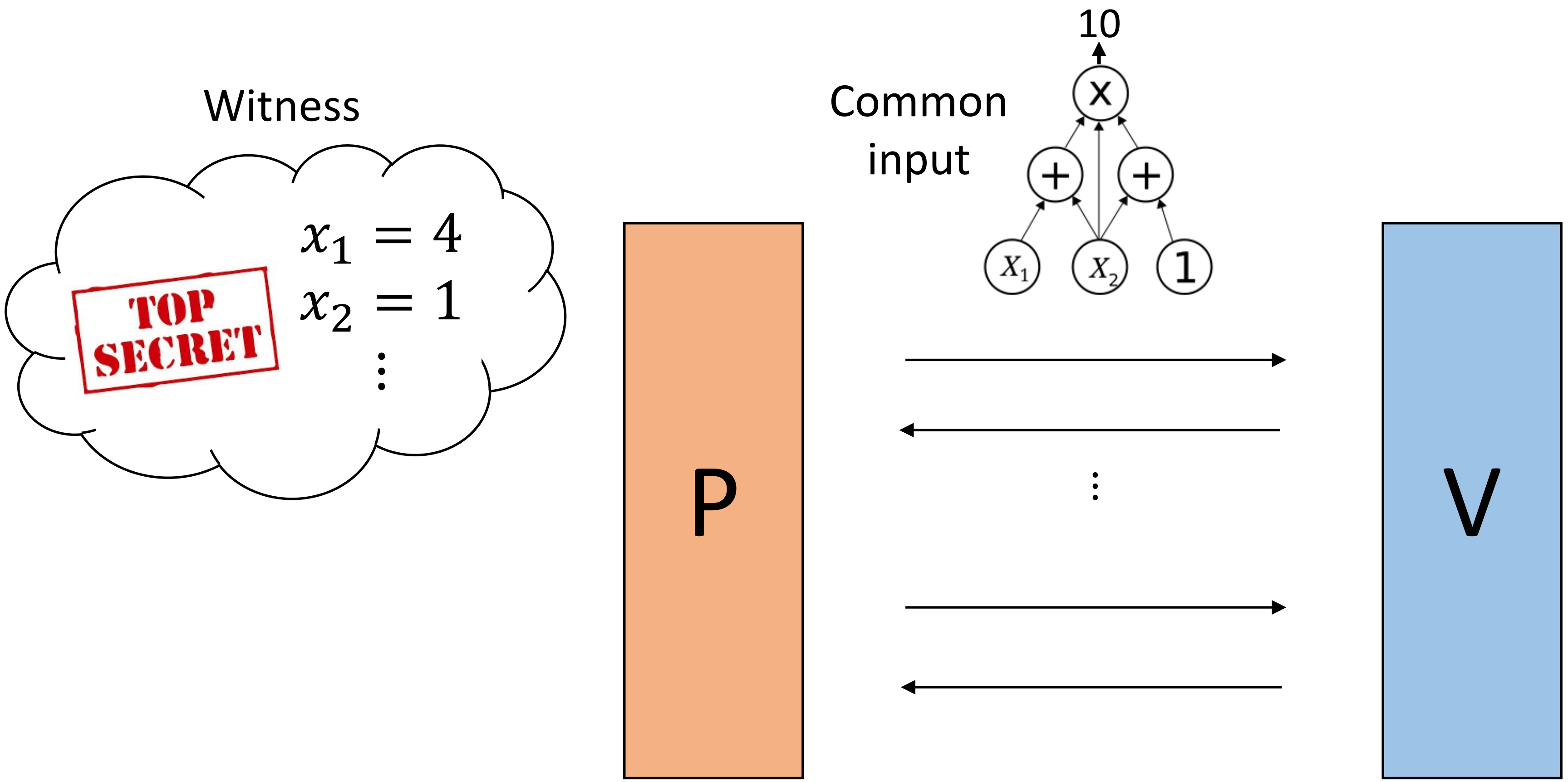

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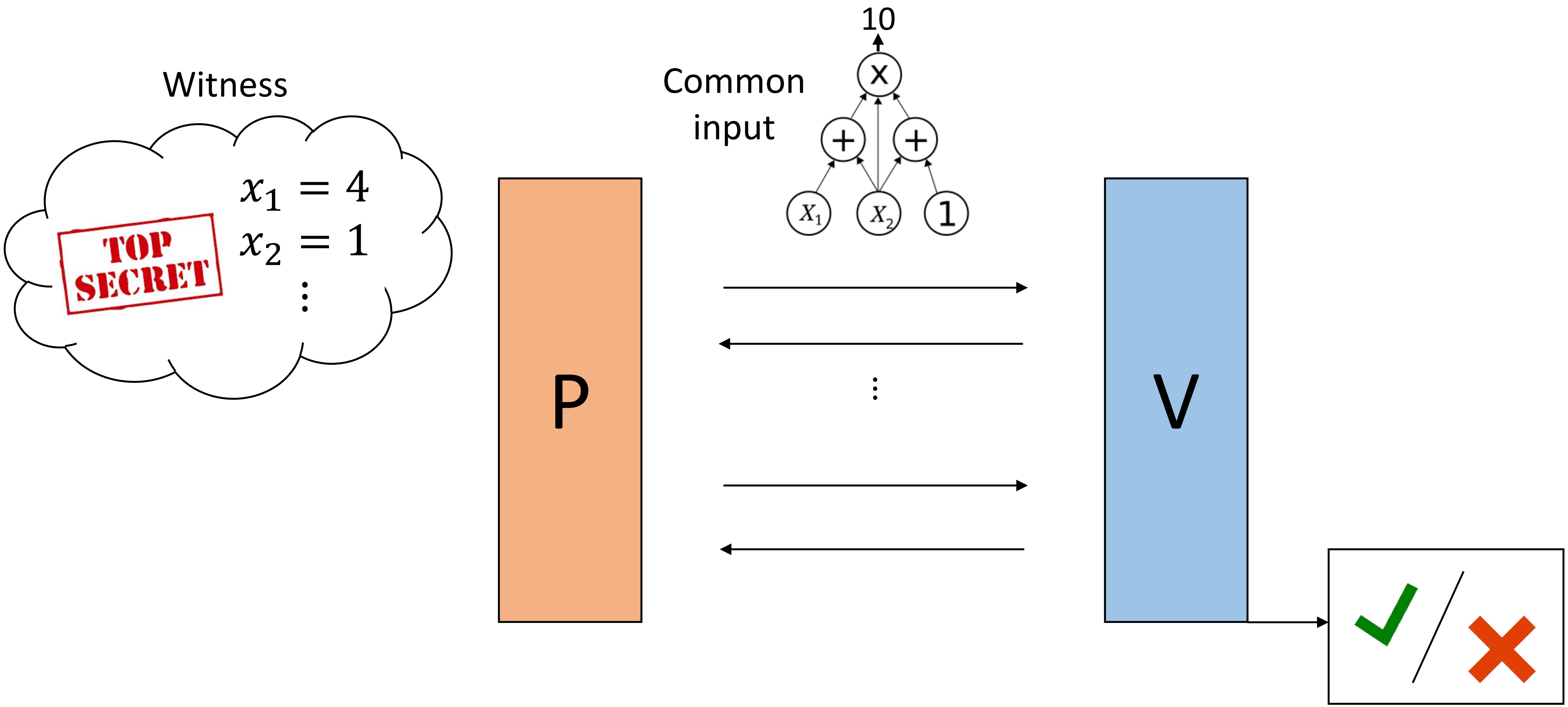

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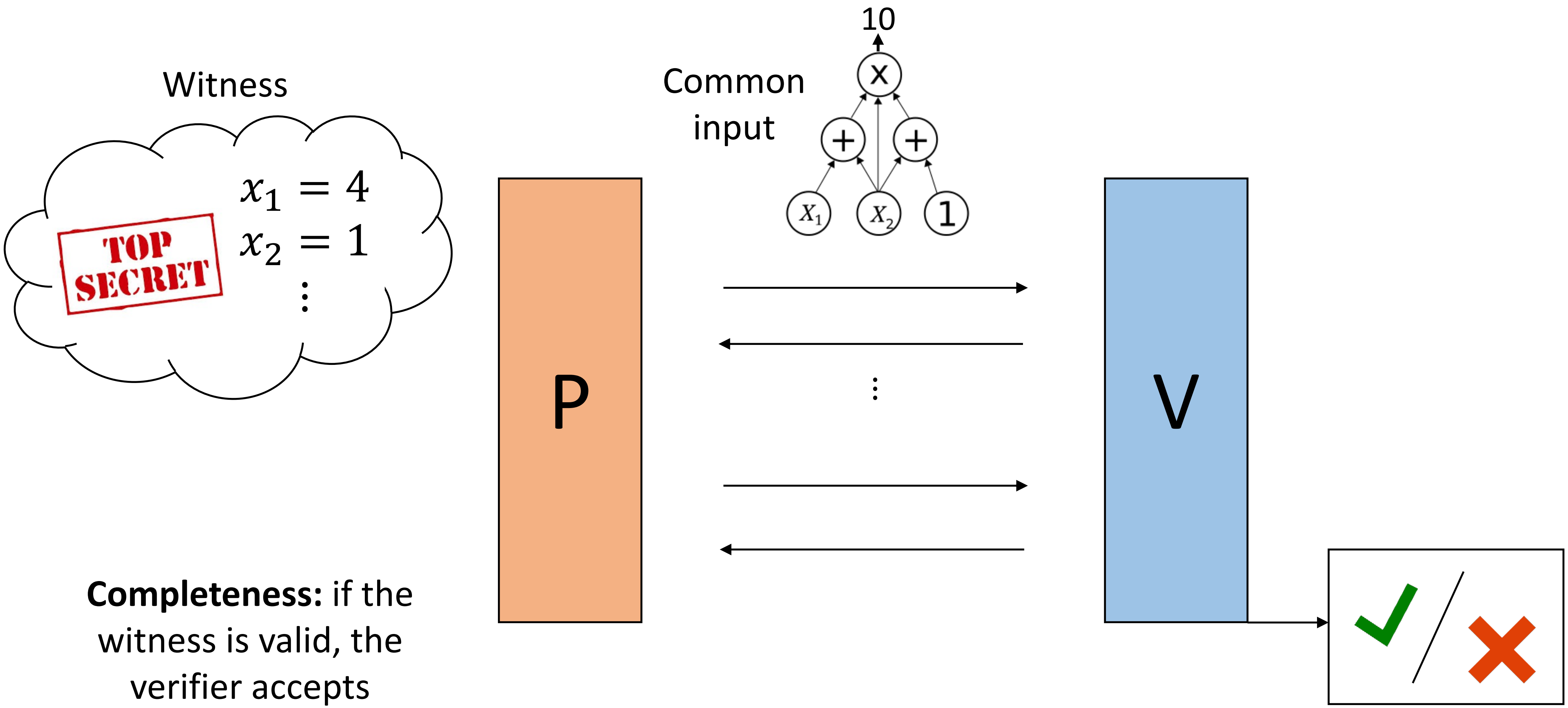
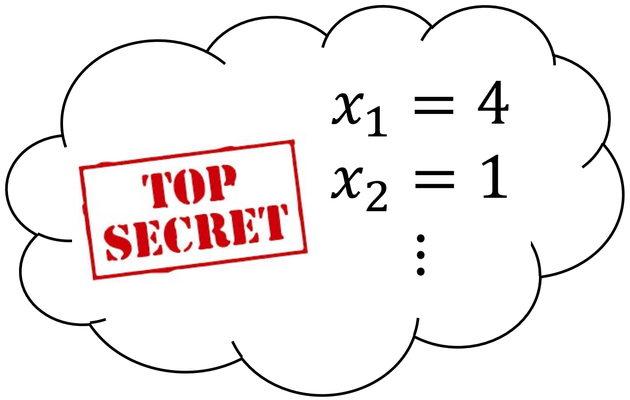


Completeness: if the witness is valid, the verifier accepts

Succinct arguments

Witness



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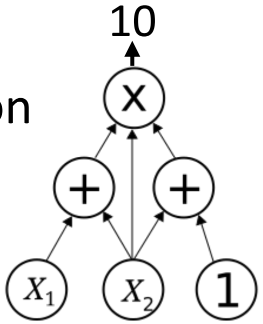
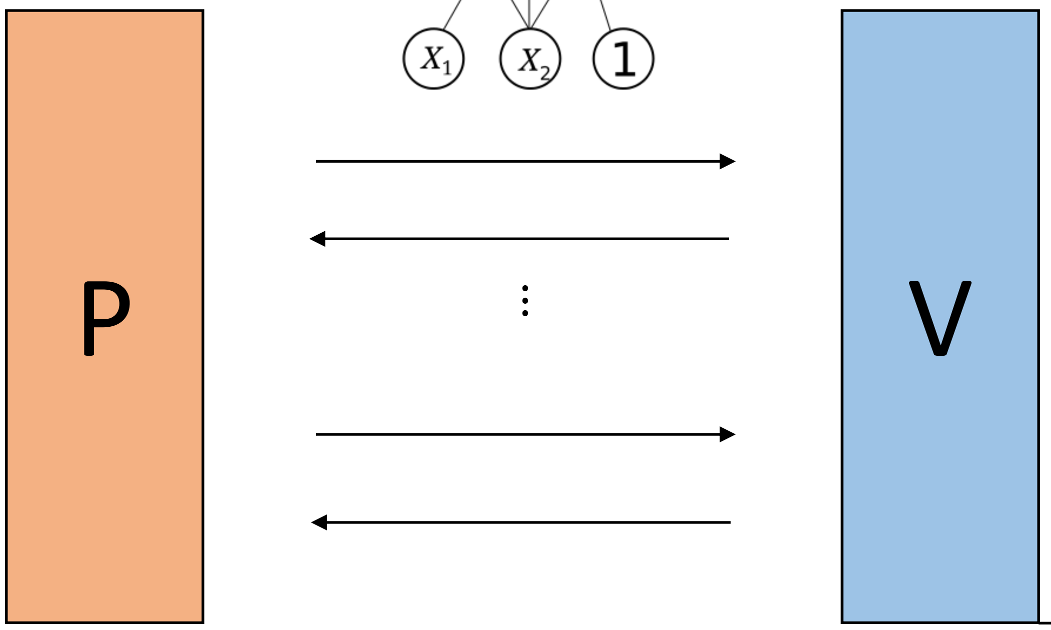
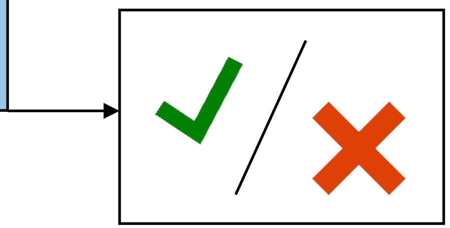




Soundness: if there is no witness, the verifier rejects

Knowledge soundness: if the prover does not know a witness, the verifier rejects



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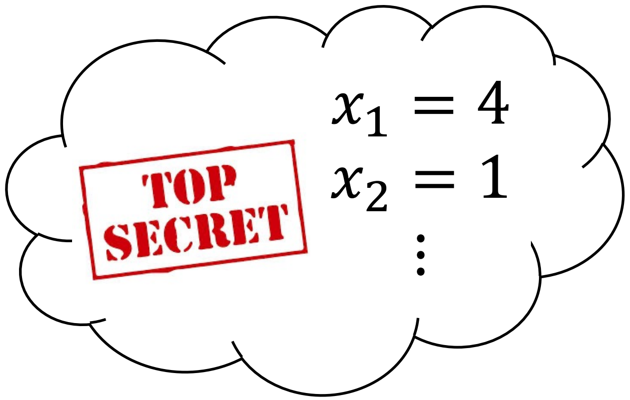

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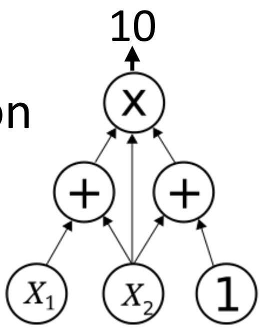

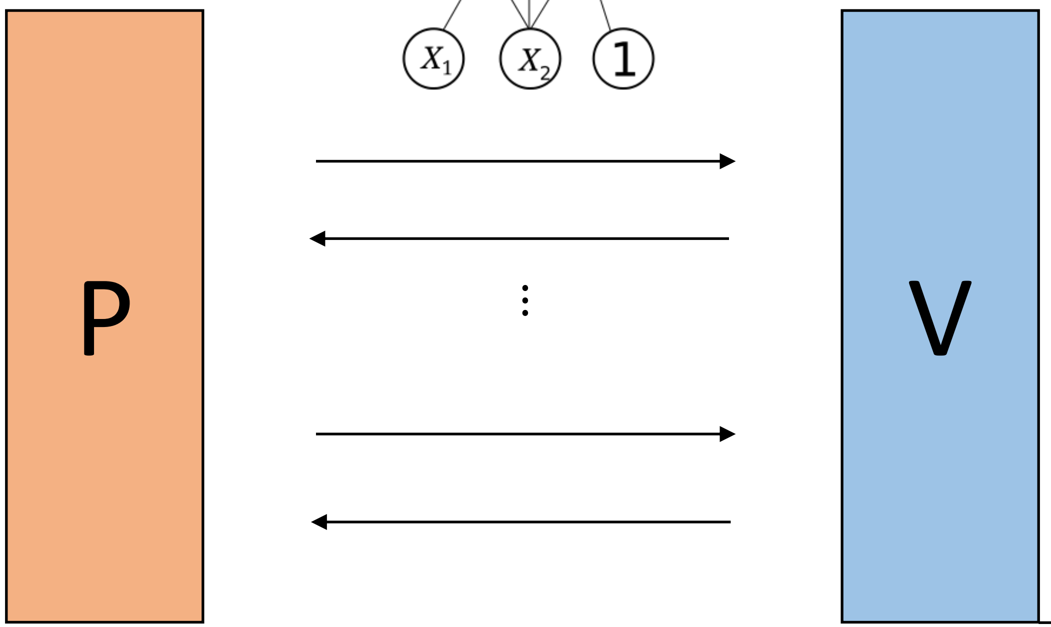

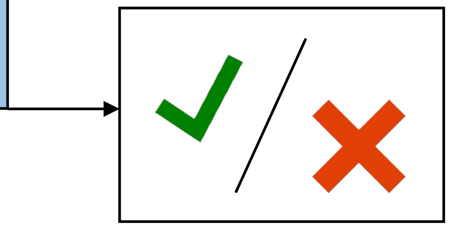

Soundness: if there is no witness, the verifier rejects

Knowledge soundness: if the prover does not know a witness, the verifier rejects

Completeness: if the witness is valid, the verifier accepts

Succinctness: the messages are much smaller than the witness

Building post-quantum succinct arguments

Building post-quantum succinct arguments

Hash-based

e.g. Aurora [BSCRSVW19]
Orion [XZS22]
Large proofs (~1MB)
Transparent

Building post-quantum succinct arguments

Pre-quantum, non-standard assumptions

e.g. [Groth16]

Tiny proofs (~1KB)
Trusted setup

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Without succinct verification

e.g. Labrador [BS23]

Quite small proofs (~50KB)
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Homomorphic cryptography

Question: can we construct **transparent**, succinct arguments from **standard** lattice assumptions?

The sumcheck protocol [LFKN92]

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Given a polynomial $p(X_1, \dots, X_\ell)$ over a field $\mathbb{F}$ and a value $u \in \mathbb{F}$,
prove that $\sum_{\underline{\omega} \in H^\ell} p(\omega_1, \dots, \omega_\ell) = u$

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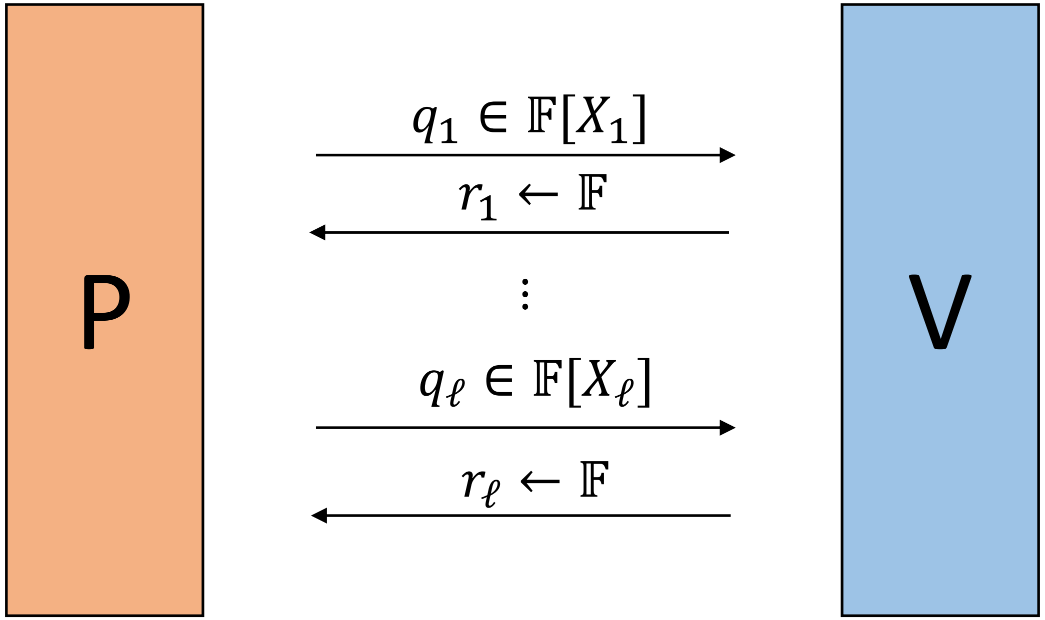
Computes polynomials
$q_i(X_i) = \sum_{\underline{\omega} \in H^{\ell-i}} p(r_1, \ldots, r_{i-1}, X_i, \omega_{i+1}, \ldots, \omega_\ell)$



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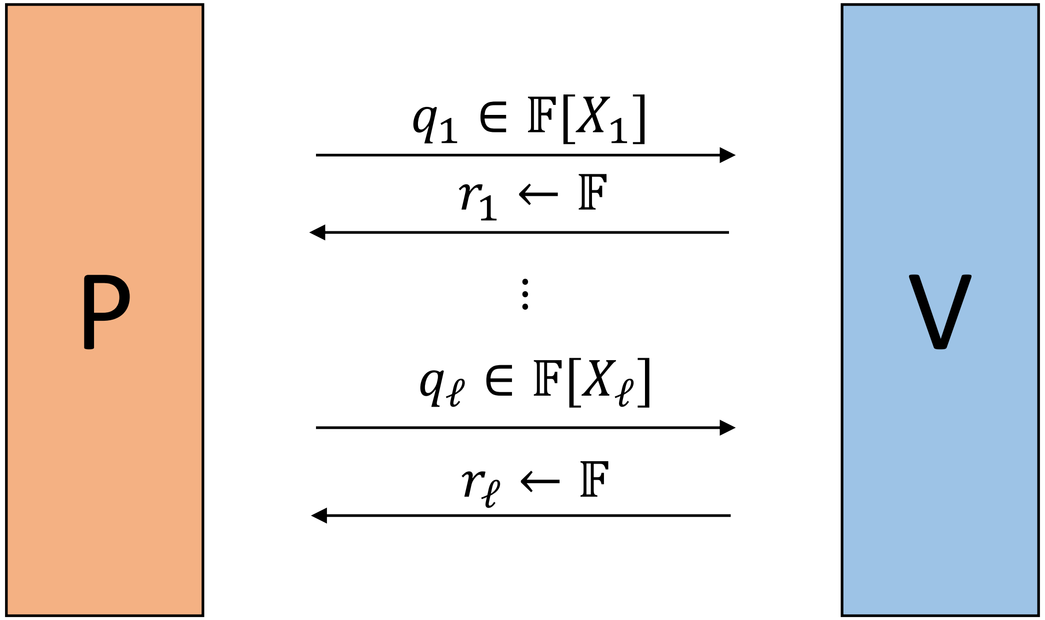

Checks that
$\sum_{\omega_1 \in H} q_1(\omega_1) = u$
$\sum_{\omega_2 \in H} q_2(\omega_2) = q_1(r_1)$
$\vdots$
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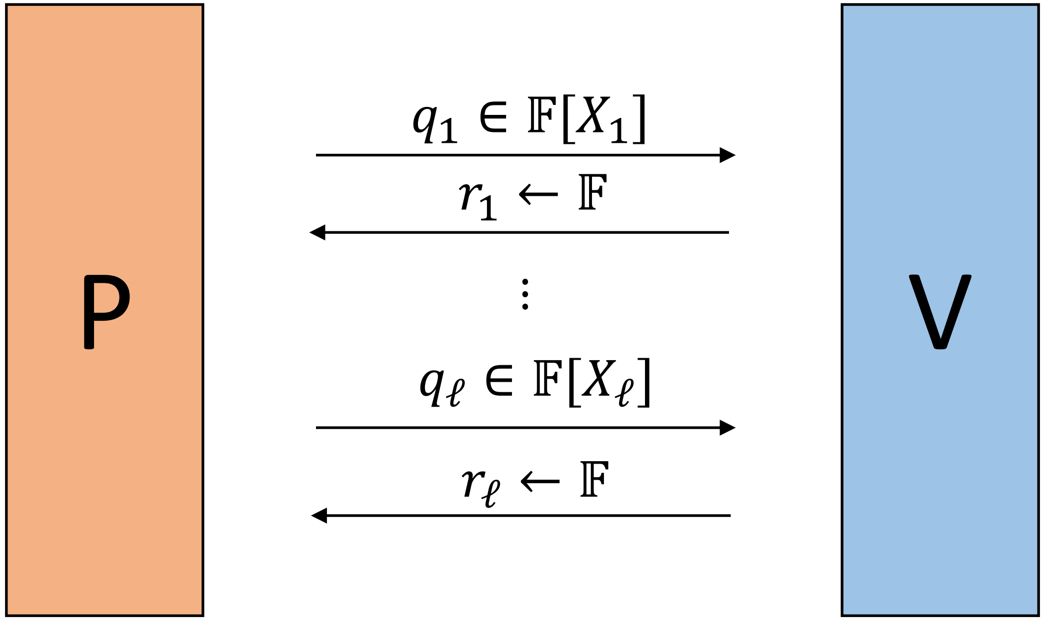

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⋮
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Evaluates p to check that
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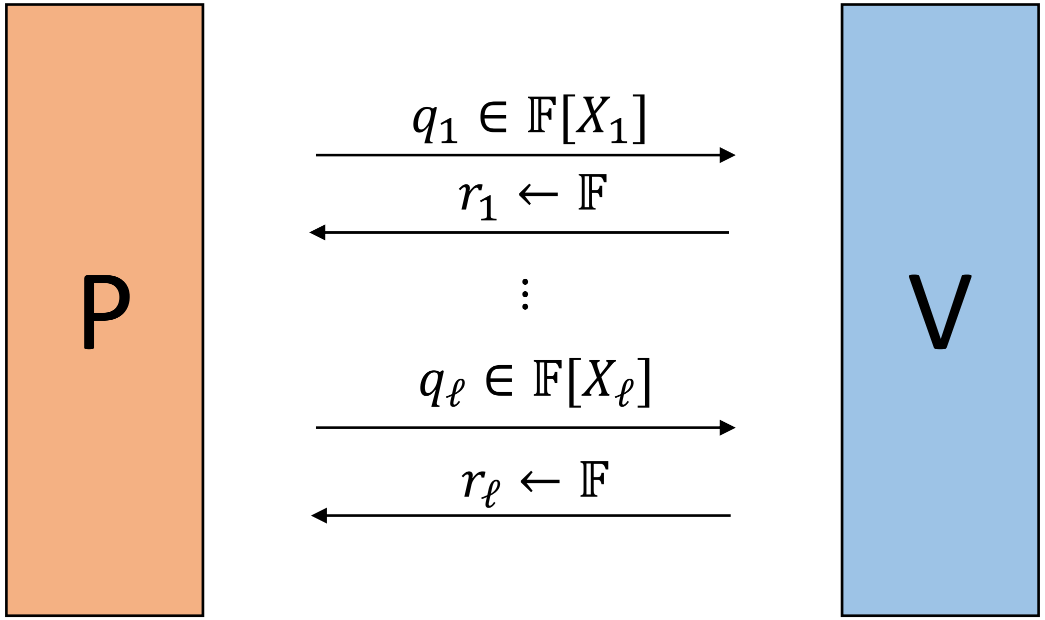

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Soundness: If $\sum_{\underline{\omega} \in H^\ell} p(\omega_1, \dots, \omega_\ell) \neq u$ then V accepts with probability at most $\frac{\ell \cdot \deg(p)}{|\mathbb{F}|}$.

The sumcheck protocol is everywhere!

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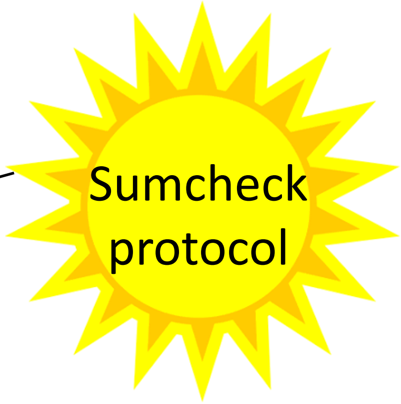

Probabilistic proofs
[BFL91,BFLS91,GKR08]

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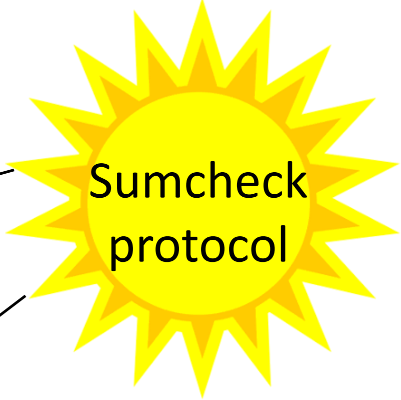

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Sumcheck-based
succinct arguments
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[CMT13], [VSBW13],
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Univariate-sumcheck-based arguments
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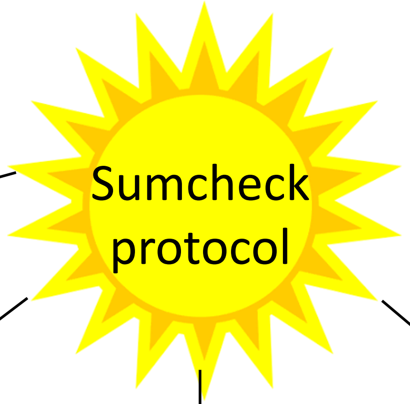

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Sumchecks for
tensor codes
[Meir13]

[RR20],
[BCG20],
[BCL20]

The sumcheck protocol is everywhere!

Sumcheck protocol

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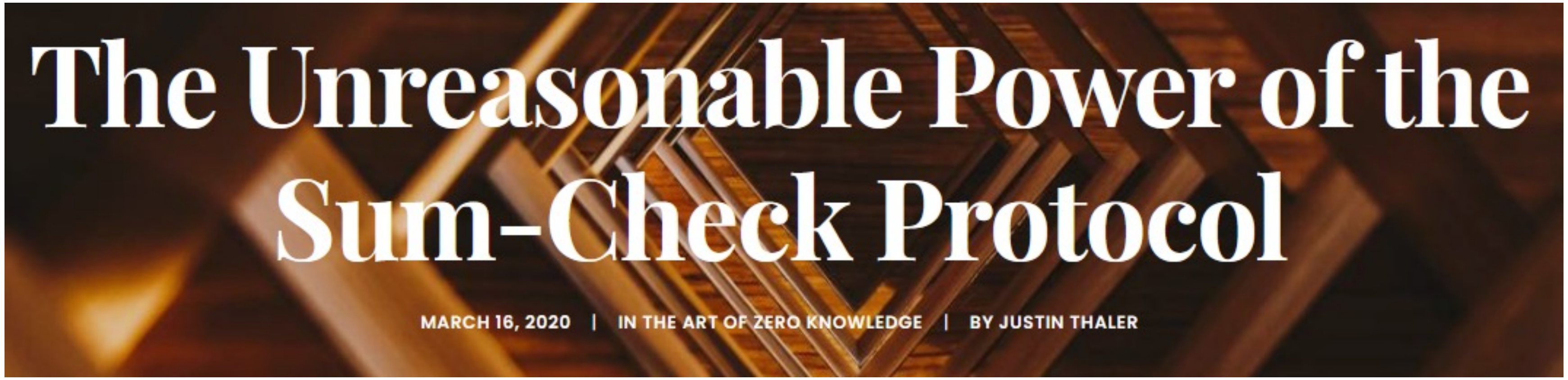

https://zkproof.org/2020/03/16/sum-checkprotocol/

Folding technique based on homomorphic enc: a separate body of work?

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Discrete-log arguments
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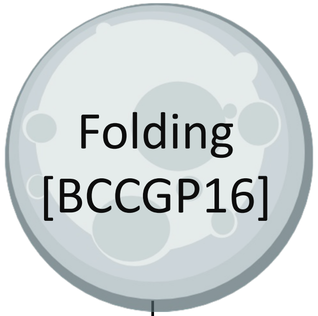

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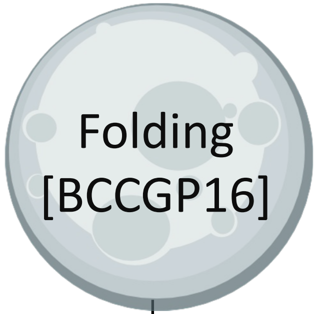

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[BLNS20], [ACK21], [LA20]

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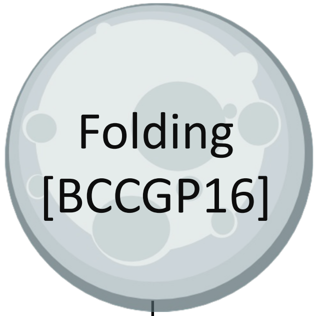

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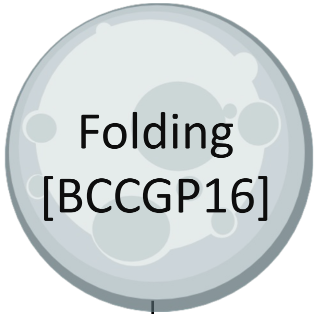

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[BBBPWM18], [PLS19],
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[BBBPWM18] implemented in Rust, Haskell, Javascript, and deployed by Blockstream, and in Monero, Mimblewimble and more…


Aim, Fire: Bulletproofs Is a Crypto Privacy Breakthrough

https://www.coindesk.com/aim-fire-bulletproofs-breakthrough-privacy-blockchains

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Results

From two bodies of work…

Sumcheck protocol

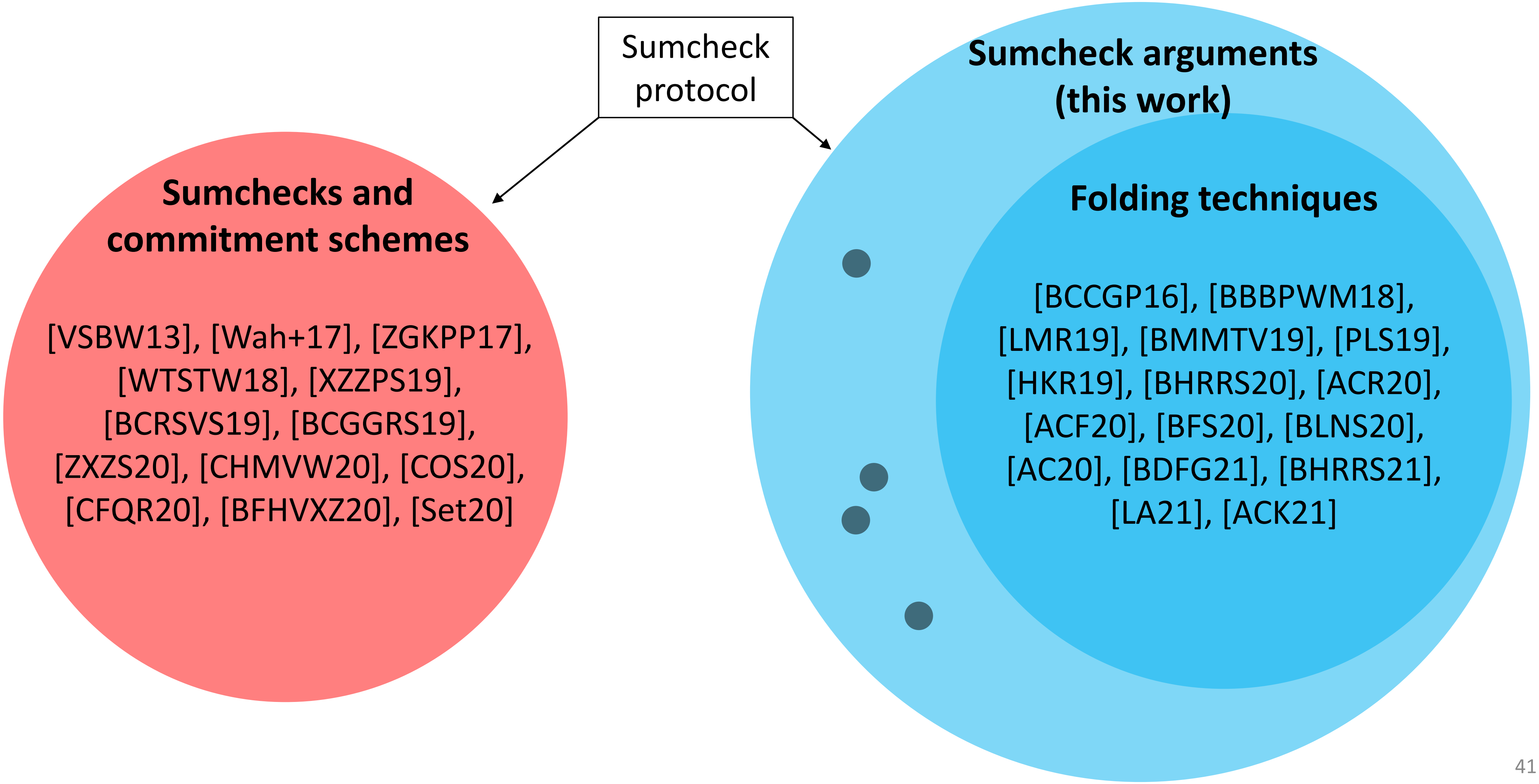
Sumchecks and commitment schemes

[VSBW13], [Wah+17], [ZGKPP17], [WTSTW18], [XZZPS19], [BCRSVS19], [BCGGRS19], [ZXZS20], [CHMVW20], [COS20], [CFQR20], [BFHVXZ20], [Set20]

Folding techniques

[BCCGP16], [BBBPWM18], [LMR19], [BMMTV19], [PLS19], [HKR19], [BHRRS20], [ACR20], [ACF20], [BFS20], [BLNS20], [AC20], [BDFG21], [BHRRS21], [LA21], [ACK21]

…to a unified perspective



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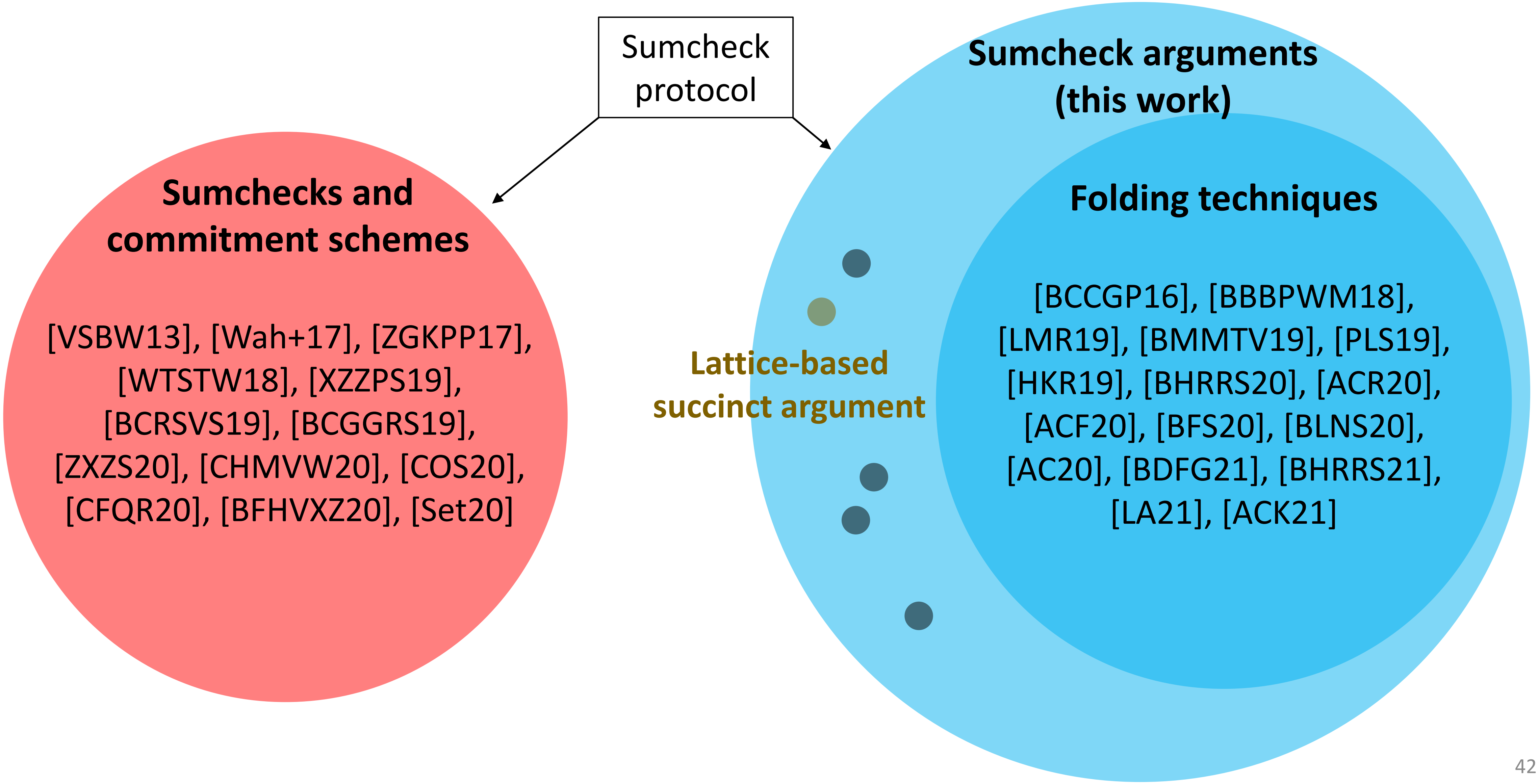
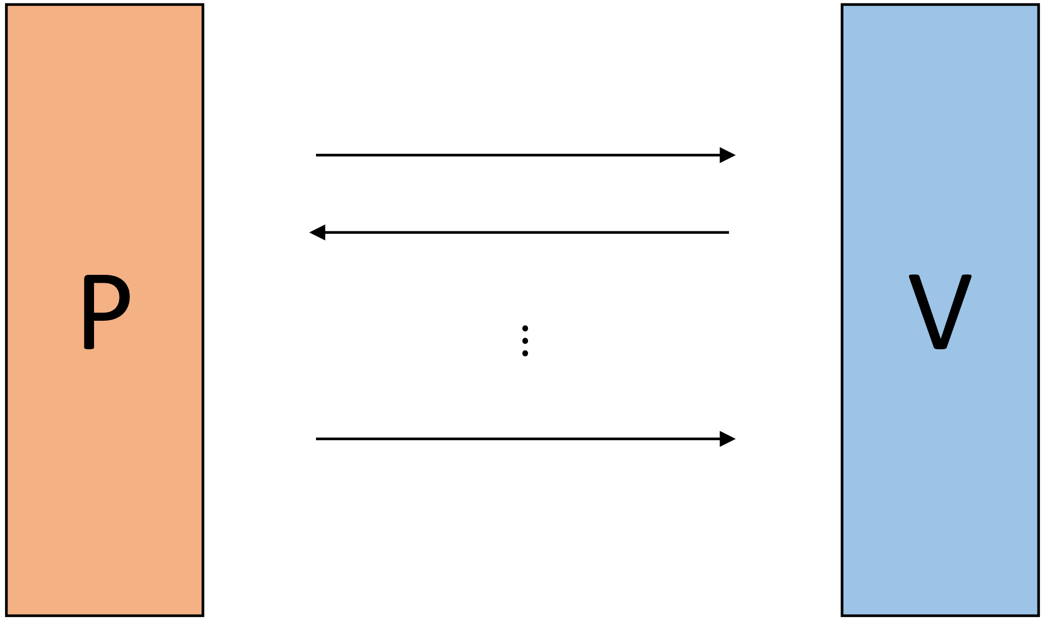


General goal:
succinct arguments for commitment openings



General goal: succinct arguments for commitment openings

Common input:
- commitment C
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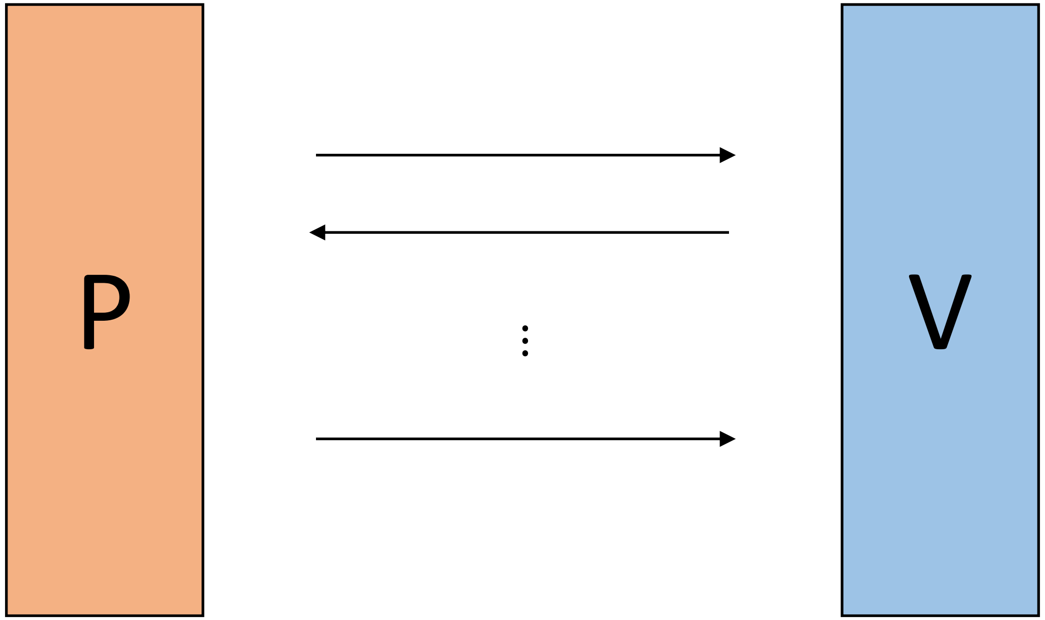

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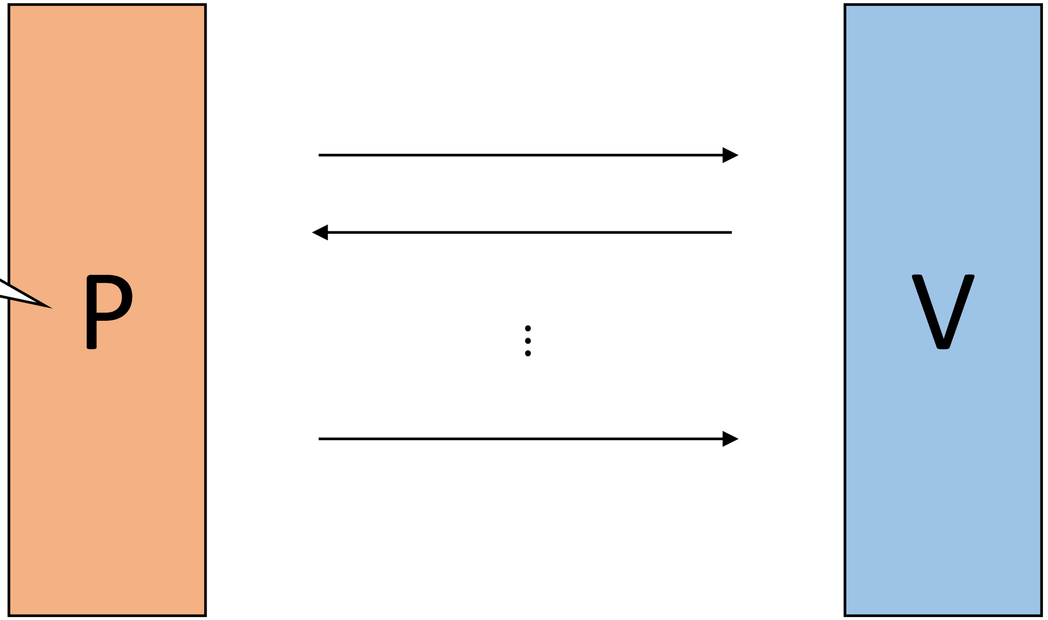

Claim: $\exists\ m$ such that $C = \mathrm{Com}(ck, m)$

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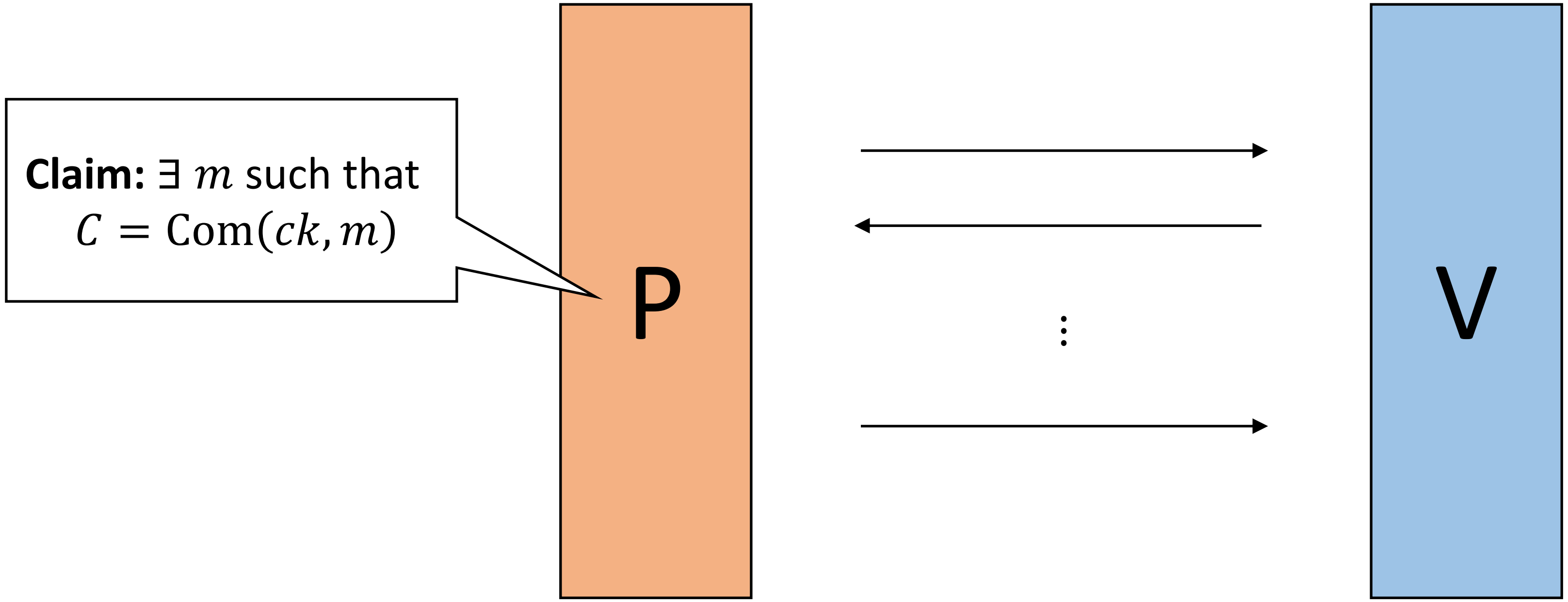

Succinctness goal:
communication $\ll |m|$

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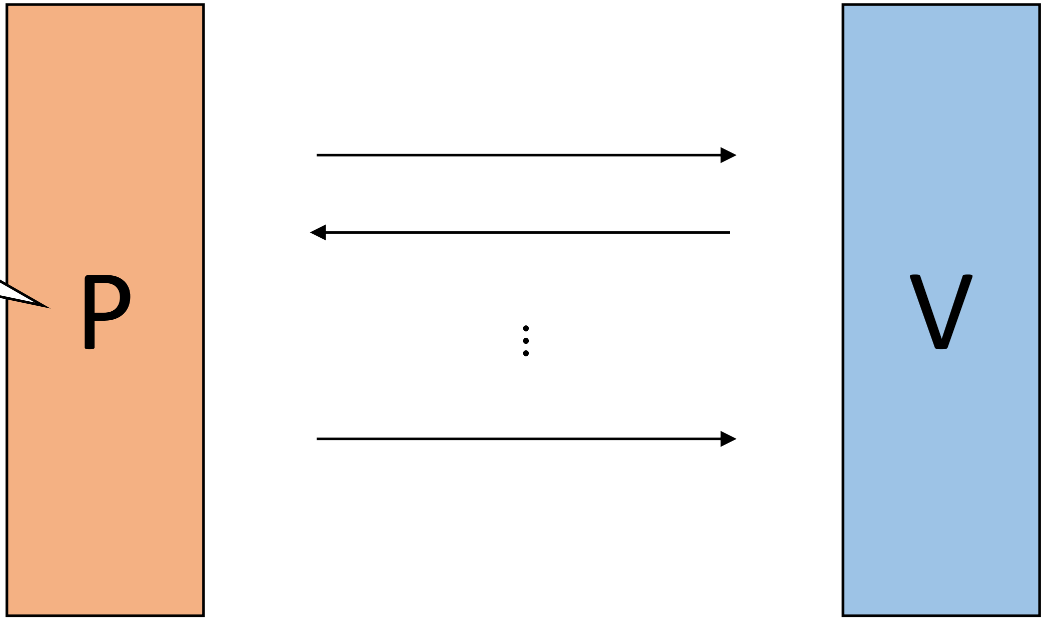

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A new notion : sumcheck-friendly commitments

Definition: A commitment scheme CM is sumcheck friendly if

$$\mathrm{Com}(ck, m) = \sum_{\omega_1,\ldots,\omega_\ell \in H} f(p_m(\omega_1, \ldots, \omega_\ell), p_{ck}(\omega_1, \ldots, \omega_\ell))$$

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commitment space $\mathbb{C}$ is an R-module

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key polynomial in $\mathbb{K}[X_1, \ldots, X_\ell]$, $\mathbb{K}$ an R-module

A new notion : sumcheck-friendly commitments

Definition: A commitment scheme CM is sumcheck friendly if

combiner function $f : \mathbb{M} \times \mathbb{K} \to \mathbb{C}$

$$\mathrm{Com}(ck, m) = \sum_{\omega_1, \dots, \omega_\ell \in H} f\left(p_m(\omega_1, \dots, \omega_\ell), p_{ck}(\omega_1, \dots, \omega_\ell)\right)$$

commitment space $\mathbb{C}$ is an R-module

evaluation points from $H \subseteq R$, R a ring

message polynomial in $\mathbb{M}[X_1, \dots, X_\ell]$, $\mathbb{M}$ an R-module

key polynomial in $\mathbb{K}[X_1, \dots, X_\ell]$, $\mathbb{K}$ an R-module

Main result: sumcheck arguments

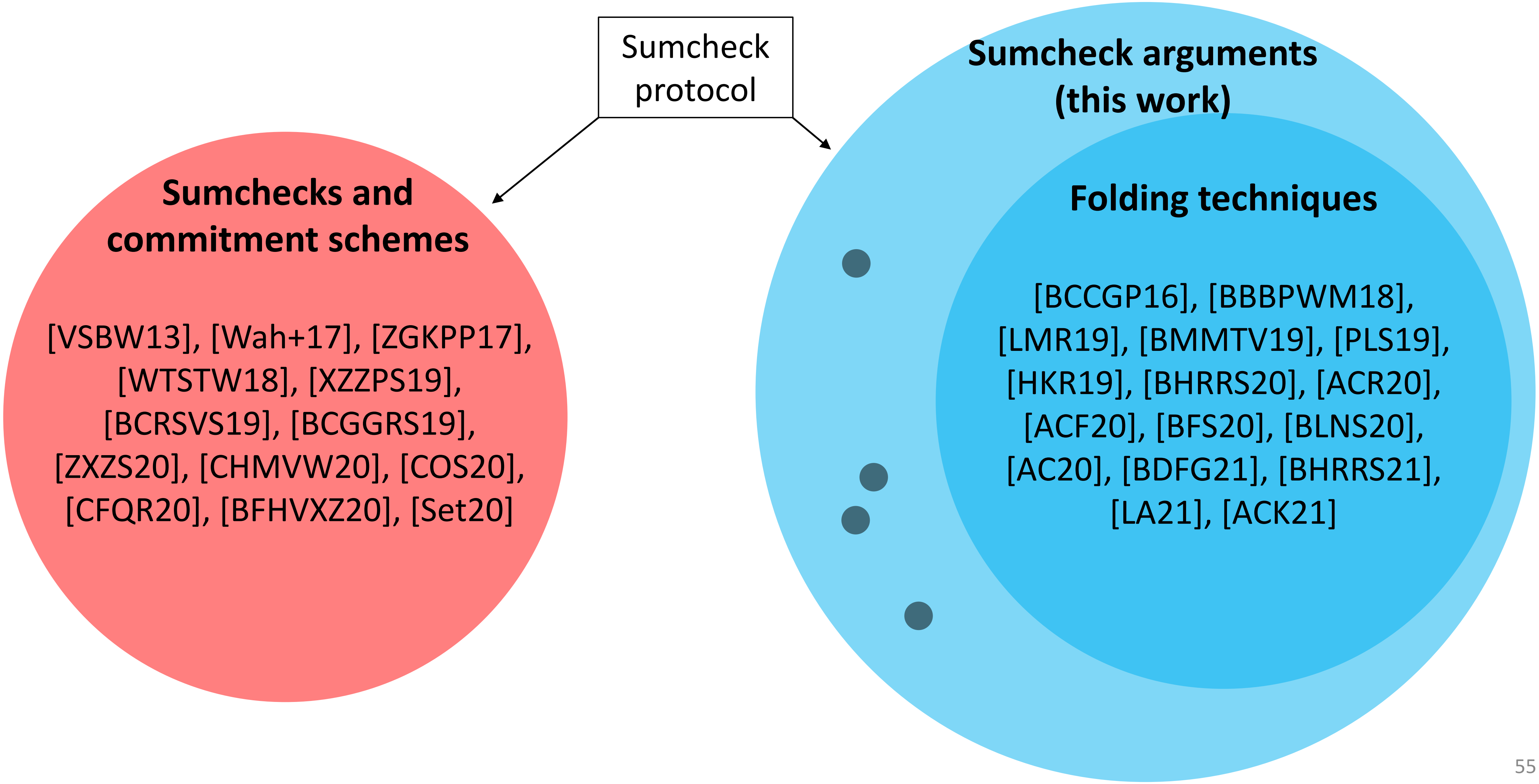
Theorem 1:

If CM is **sumcheck-friendly** and **invertible**. The sumcheck protocol applied to

$$p(X_1, \dots, X_\ell) = f\big(p_m(X_1, \dots, X_\ell), p_{ck}(X_1, \dots, X_\ell)\big) \in \mathbb{C}[X_1, \dots, X_\ell]$$

(with one extra verifier check) is a succinct argument of knowledge with communication $\ell \cdot \deg(p)$

Application: succinct arguments for NP



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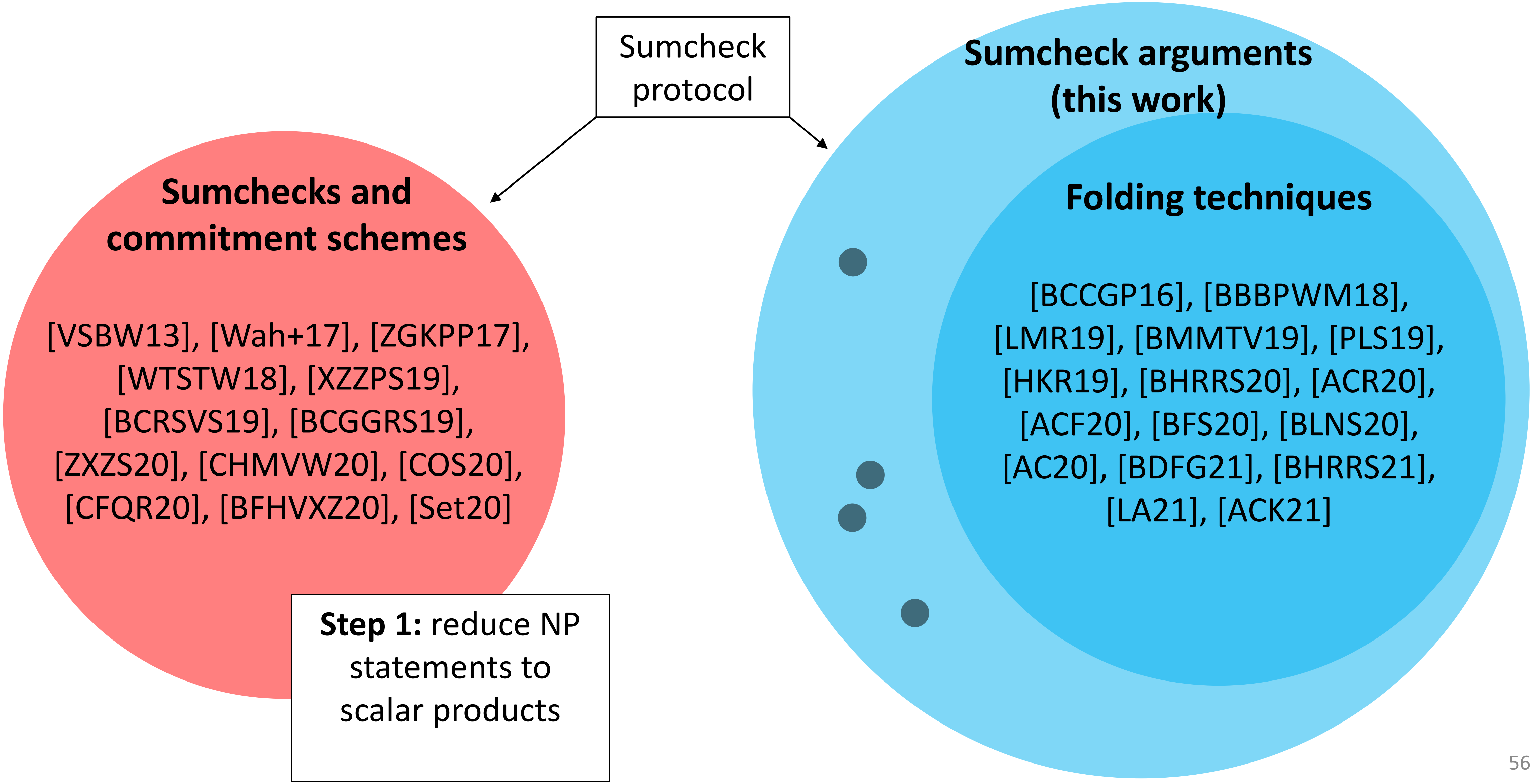

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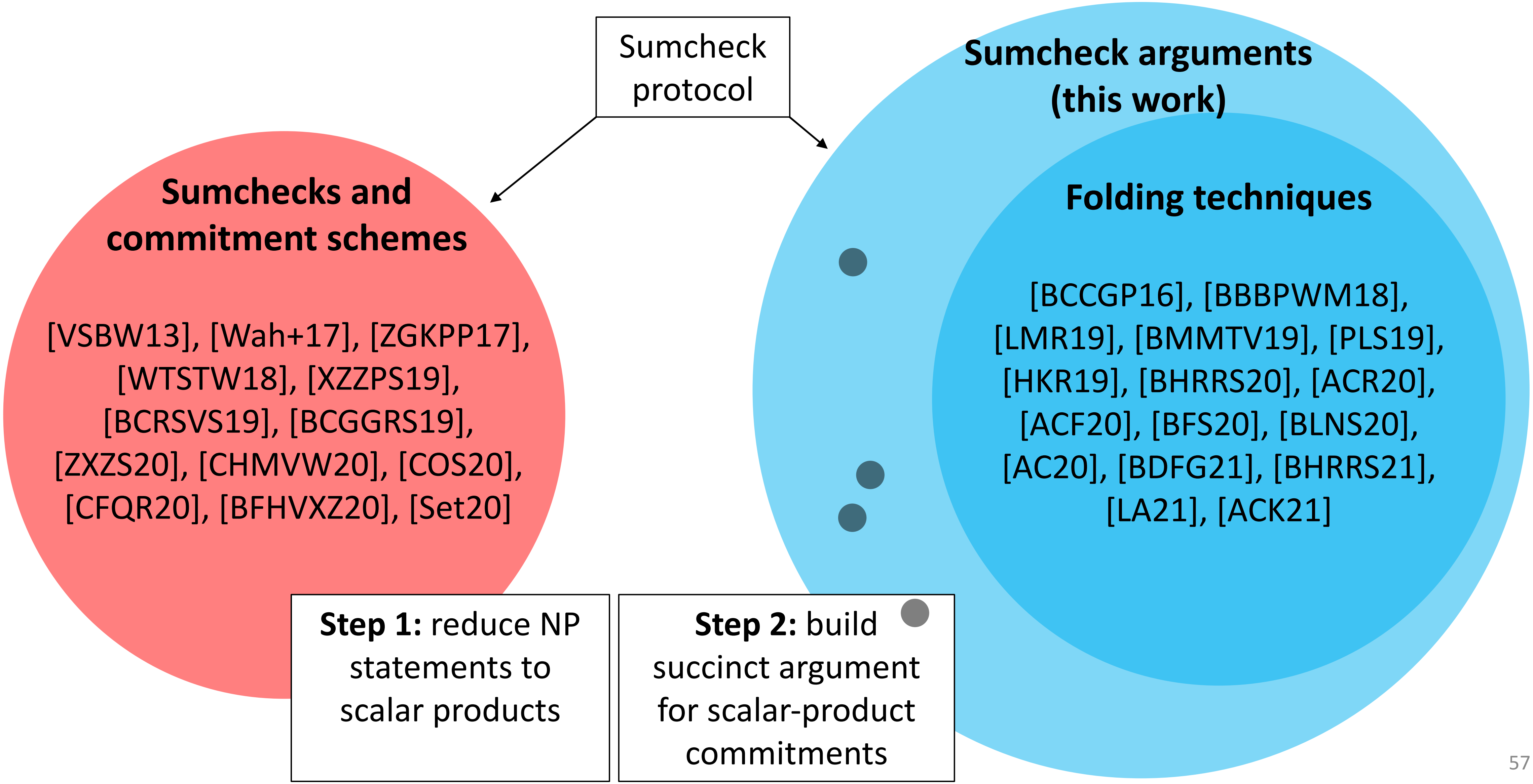

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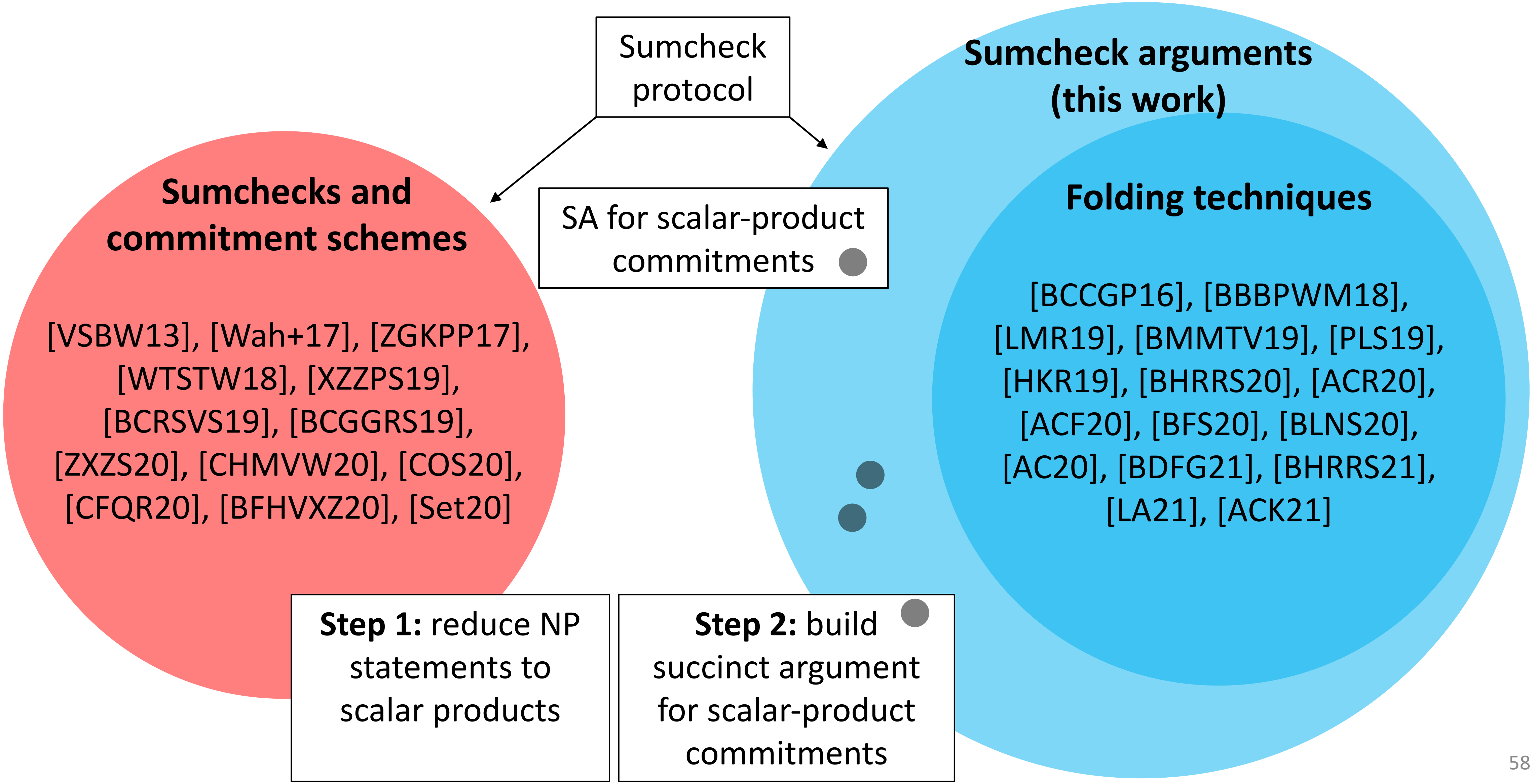

Lattice-based succinct arguments for NP

[Bootle Chiesa **Sotiraki** '21]

Corollary: Assuming SIS is hard over $R_q \coloneqq \mathbb{Z}_q[X]/\langle X^d + 1\rangle$ and $p \ll q$ primes, there is a *zero-knowledge* succinct argument of knowledge for NP with

R1CS Ring	Prover time	Verifier time	Proof size
R_p	$O(n)$ ops in R_p, R_q	$O(n)$ ops in R_p, R_q	$O(\log n)$ elems of R_q

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Techniques

Sumcheck argument for Pedersen

Common input:

- commitment $C \in \mathbb{G}$
- key $\underline{G} \in \mathbb{G}^n$

Claim: $\exists \underline{a} \in \mathbb{F}^n$ s.t. $C = \langle \underline{a}, \underline{G} \rangle$

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Succinct verification via delegation [Bootle Chiesa **Sotiraki** '23]

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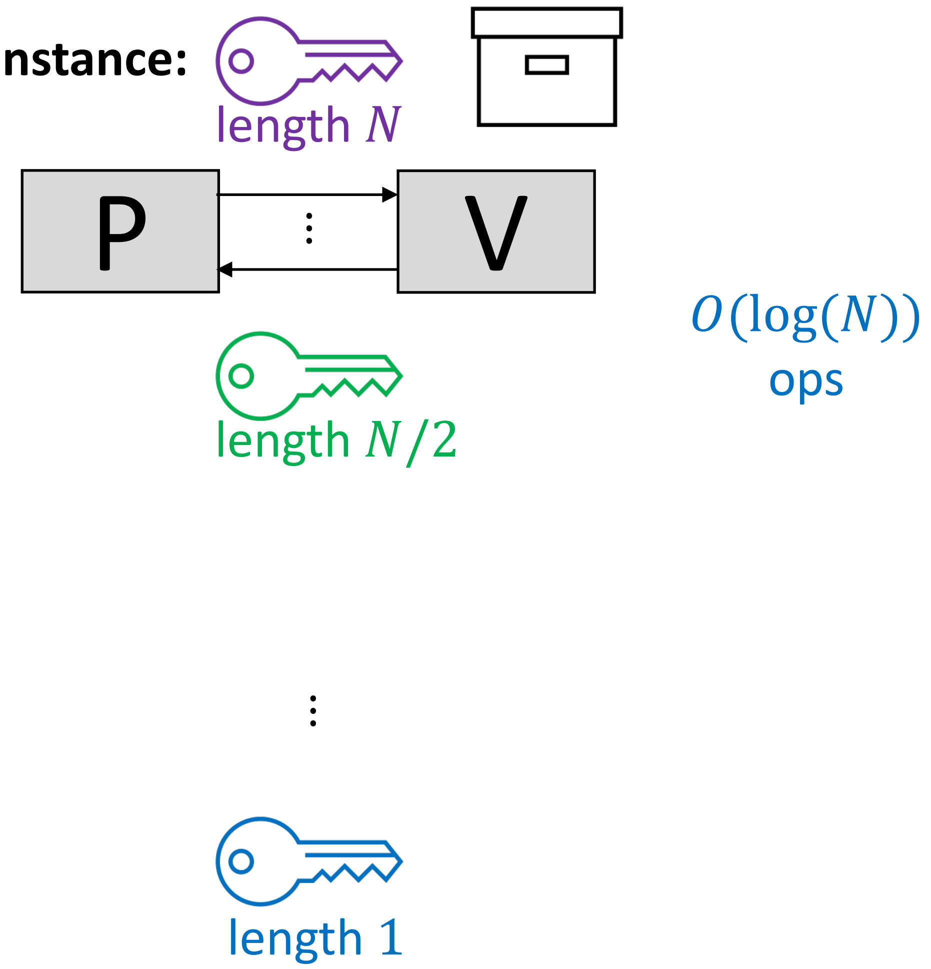
Witness: p

Instance:



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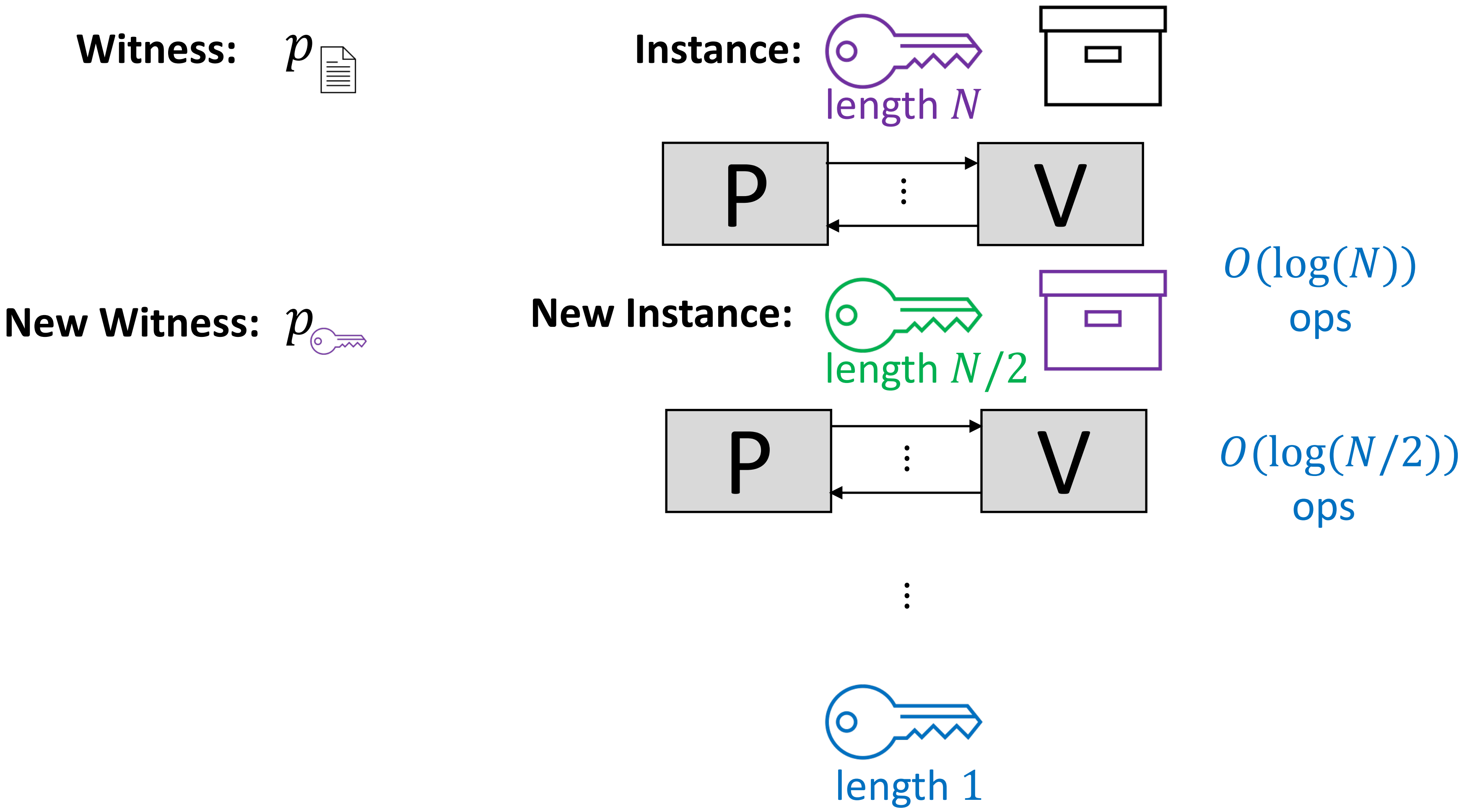

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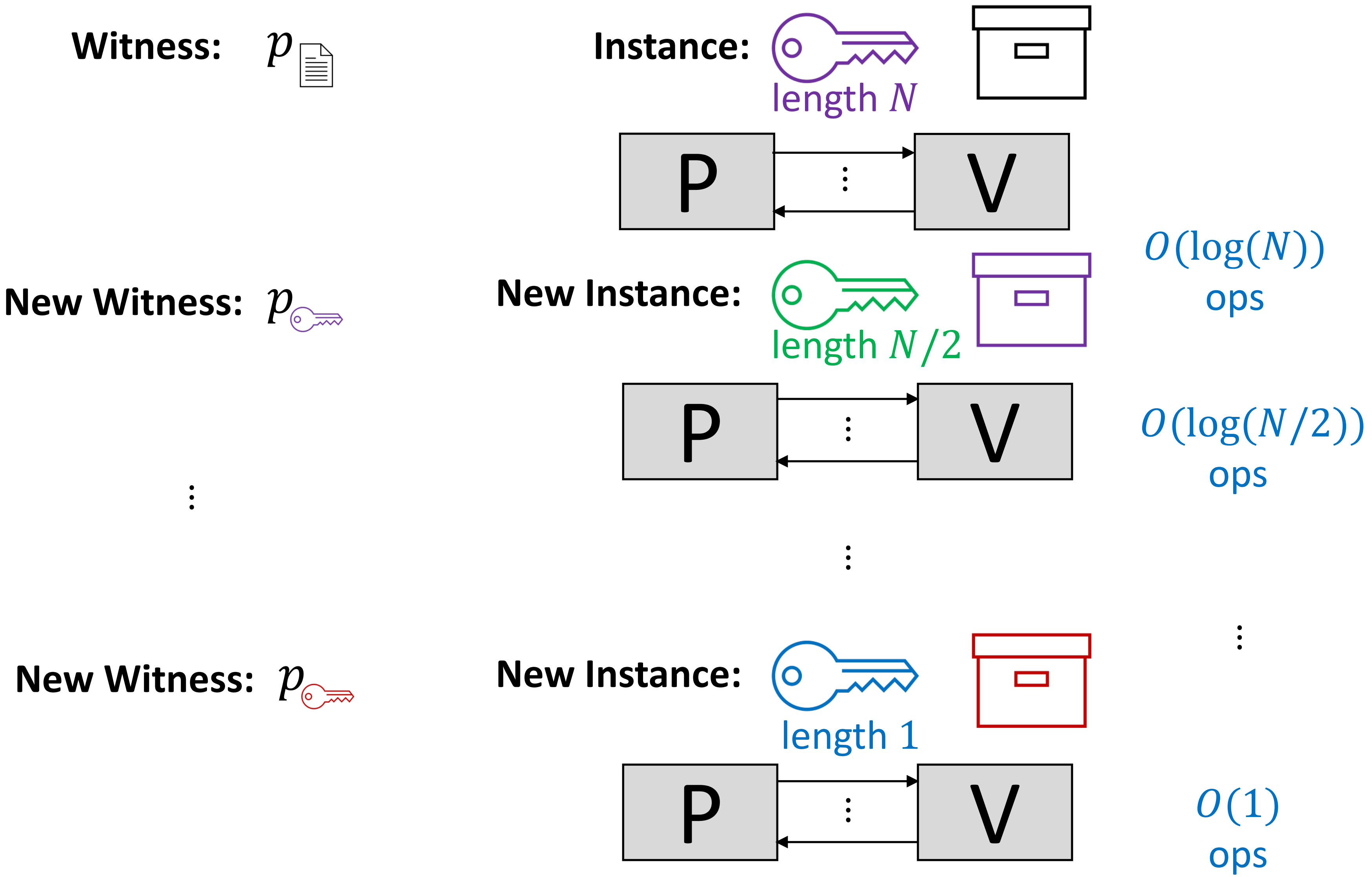

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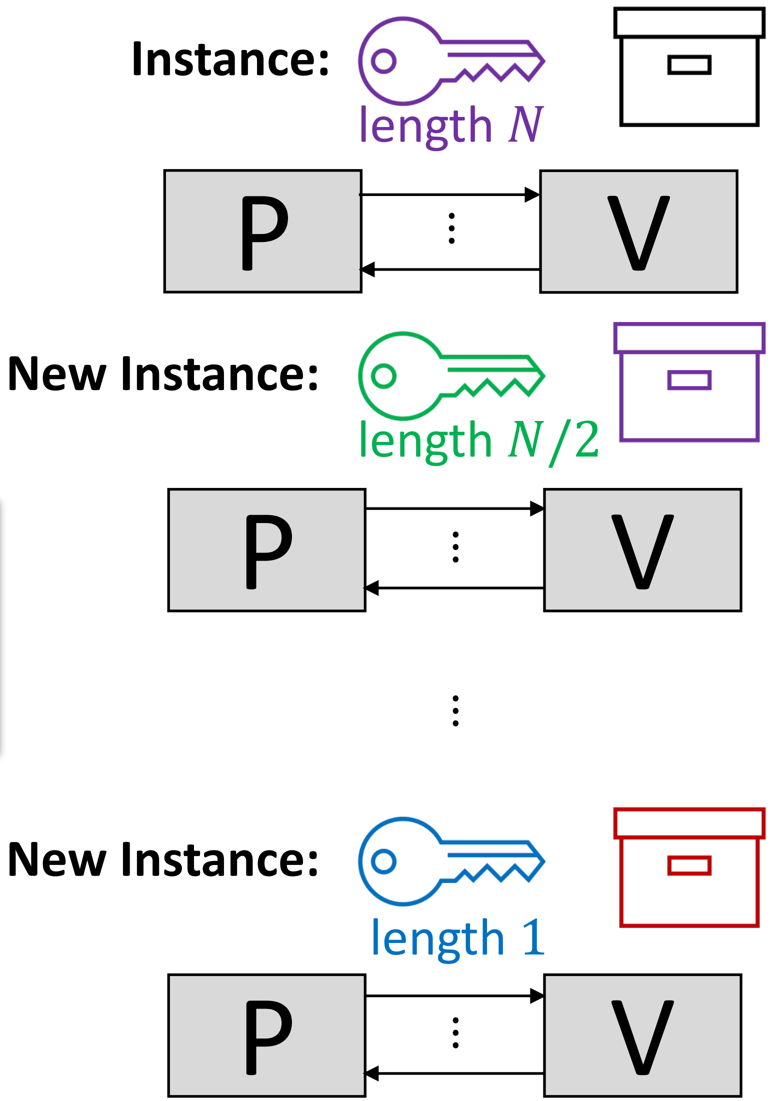

Generalises approach from [Lee21], [Thaler] beyond pairings

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What kind of soundness? **Knowledge soundness**

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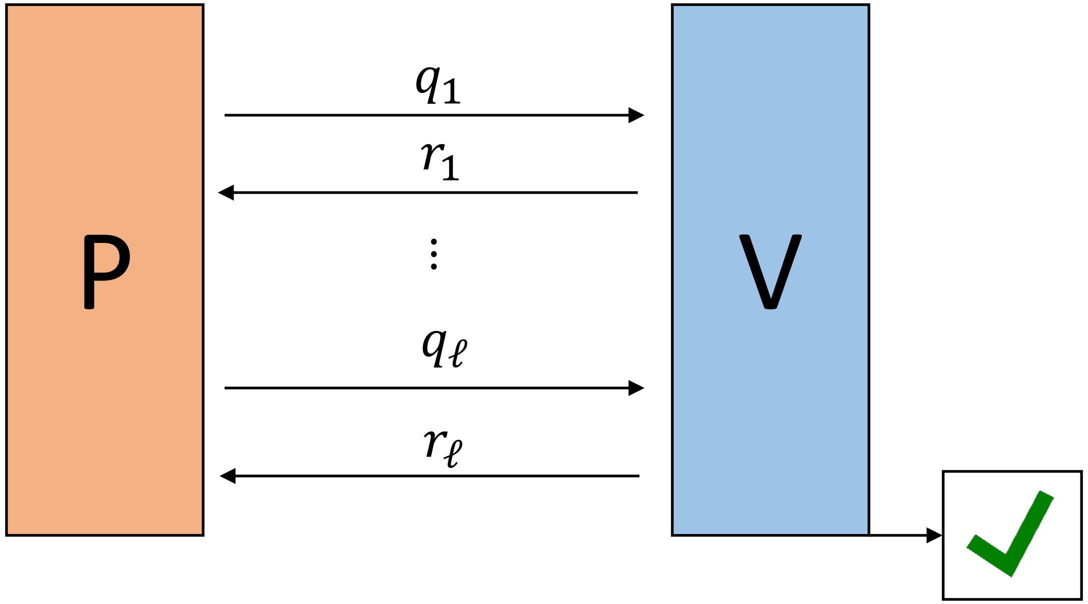

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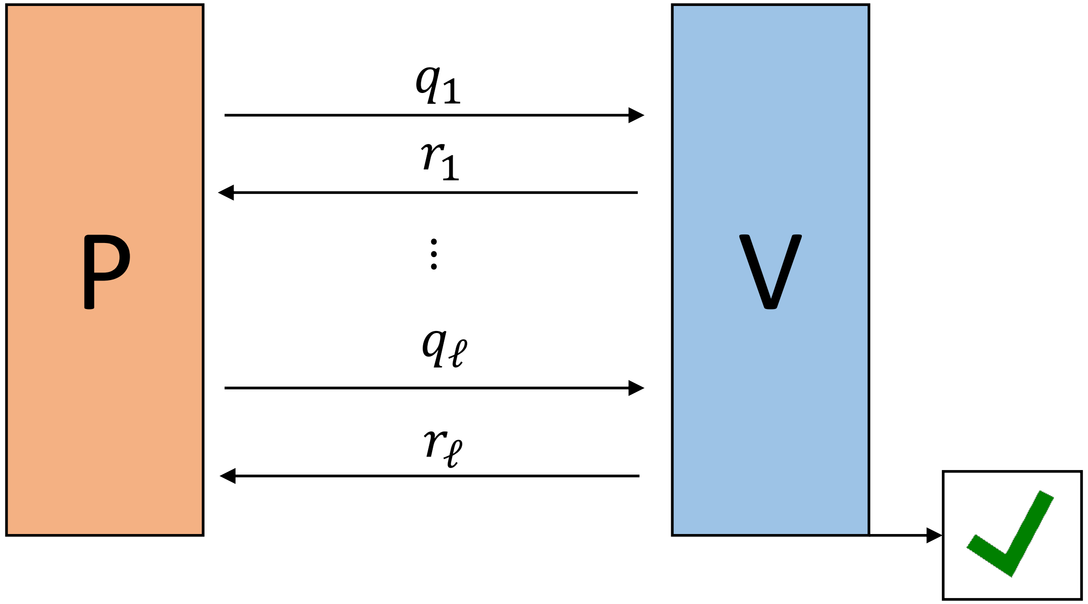

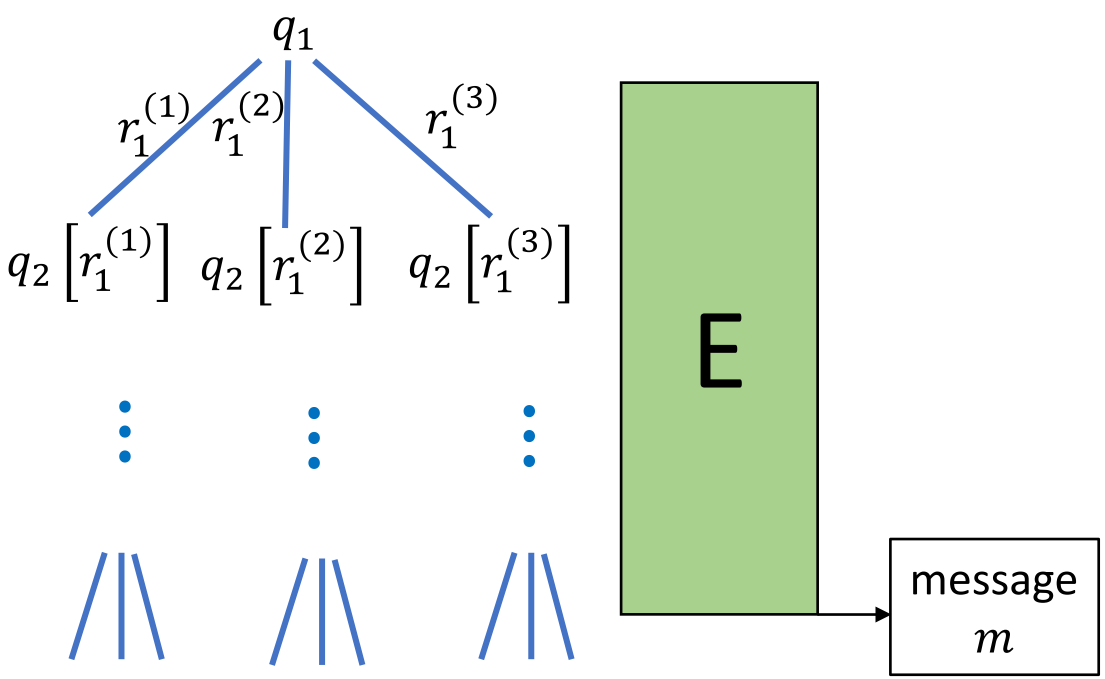

From groups to rings

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Lattices and groups of unknown order?

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Lattices and groups of unknown order?

Solution: an abstraction for mathematical structures where folding techniques can work

From groups to rings:
bilinear modules

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R-module M: generalization of vector space over rings

From groups to rings: bilinear modules

R-module M: generalization of vector space over rings

Assumption	Messages	Keys	Commitments	Ideal
BRA	small M_L	M_R	M_T	I
DLOG	$\mathbb{F}_p$	$\mathbb{G}$	$\mathbb{G}$	$\{0\}$
DPAIR[AFGHO10]	$\mathbb{G}_1$	$\mathbb{G}_2$	$\mathbb{G}_T$	$\{0\}$
UO [BFS20]	small $\mathbb{Z}$	$\mathbb{G}$	$\mathbb{G}$	$n\mathbb{Z}$ for suitable small n
RSIS [Ajtai94]	small R_q	R_q^d	R_q^d	$n\mathbb{Z}$ for suitable small n

Takeaways

- There are lattice-based transparent, succinct arguments
- Many commitment schemes are sumcheck friendly
- We can recast many different cryptographic settings as bilinear modules

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Thanks!