Algorithmic Game Theory - Part 2

Online Mechanism Design

Nikolidaki Aikaterini
aiknikol@yahoo.gr

Corelab,
NTUA

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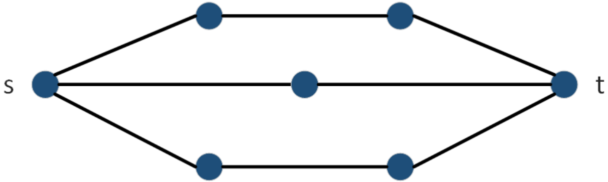
Overview

1. Mechanism Design
   - Truthful Mechanisms
2. Scheduling Problems
   - Related Machines
   - Unrelated Machines
3. Online Mechanisms
   - Dynamic Auction with Expiring Items
   - Secretary Problem
   - Adaptive Limited-Supply Auction
4. Procurement Auctions
   - Frugal Path Mechanisms
   - Budget Feasible Mechanisms
   - Learning on a Budget: Posted Price Mechanisms

Frugal Path Auctions

A problem of finding frugal mechanism

- To buy an inexpensive s-t path
- Each edge is owned by a selfish agent.
- The cost of an edge is known to its owner only.
- Goal: to investigate the payments the buyer to get a path



- A possible solution: VCG mechanism, which pays a premium to induce the edges to reveal their costs truthfully
- Goal: to design a mechanism that selects a path and induces truthful cost revelation without paying such a high premium

Frugality

- Ordinary Vickrey procurement auction: frugal?

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- VCG shortest path mechanism: frugal?
  - * NO!

► Some Instances: Mechanism pays $\Theta(n)$ times the actual cost of path, even if there is an alternate path available that costs only $(1+\epsilon)$

Frugality

We want to design mechanisms that AVOID LARGE OVERPAYMENTS!

Reasonable Mechanism Properties

- Path Autonomy: Given any b_{-P} bids of all edges outside P, there is a bid b_P such that P will be chosen
- Edge Autonomy: For any edge e, given the bids of the other edges, e has a high enough bid that will ensure that no path using e will not win
- Independence: If path P wins, and an edge $e \notin P$ raises its bid, then P will still win
- Sensitivity: Let P wins and Q is tied with P. Then increasing b_e for any $e \in P - Q$ or decreasing b_e for any $e \in Q - P$ cause P to lose

Definition

Assume path P wins. if there is an edge e such that arbitrarily small change in e's bid cause another path Q to win. Then P and Q are tied.

Min Function Mechanisms

Definition

A mechanism is called a Min Function Mechanism function if it defines for every s-t path P, a positive real valued function f_P of the vector of bids b_P, such that:

- $f_P(b_P)$ is continuous and strictly increasing in b_e, $\forall e \in P$
- The mechanism selects the path with lowest $f_P(b_P)$
- $\lim_{b_e \to \infty} f_P(b_P) = \infty$, $\forall e \in P$
- $\lim_{b_P \to 0} f_P(b_P) = 0$

* Note: Mechanism evaluates each function & select the path with the lowest function value

* A mechanism is truthful only if it has the thresold property

Min Function Mechanisms

Theorem

The min function path selection rule yields a **truthful mechanism**

Proof Sketch:

- *Path selection rule is monotone: if P is currently winning & edge $e \notin P$, then $f_P(b_P)$ is the minimum function value. Raising b_e & $e \in Q \Rightarrow$ Raising $f_Q(b_Q) \Rightarrow Q$ loses*
- *Every edge in the winning path has a threshold bid: $e \notin P$, f_P is minimum, and T_{b_e} the largest bid, $e \in Q$, beyond $T \Rightarrow P$ wins*

Theorem

Min function mechanism satisfies the **edge** *and* **path autonomy**, **independence** *and* **sensitivity** *property*

Proof Sketch:

P.A: *follows from* $\lim_{b_P \to 0} f_P(b_P) = 0$ *with positive values*

E.A: *follows from* $\lim_{b_e \to \infty} f_P(b_P) = \infty$ *with increasing functions*

Ind: *follows from f_P are strictly increasing & unaffected by edges not on P*

Sens: *follows from $f_P(b_P)$ is continuous and strictly increasing*

Characterization Results

Theorem

If a graph G contains the edge s-t, then any truthful mechanism for the s-t path selection problem on G that satisfies the **independence**, **sensitivity** *and* **edge** *and* **path autonomy** *properties is a min function mechanism*

Theorem

If a graph G consists of some connected graph including nodes s and t, plus two extra s-t path that are **disjoint** *from the rest of graph, then any truthful mechanism for the s-t path selection problem on G that satisfies the* **independence**, **sensitivity** *and* **edge** *and* **path autonomy** *properties is a min function mechanism*

Costly Example for Min-Function Mechanisms

- Let L cost of the winning path and k=♯edges
- Let b_P^i vector of bids along P and each edge bid $\frac{L}{|P|}$, except i-th bids $\frac{L}{|P|} + \epsilon L$. Similarly, the bids of path Q.
- w.l.o.g $f_Q(b_Q^1) = \max \left\{ f_P(b_P^1), ..., f_P(b_P^{|P|}), ..., f_Q(b_Q^1), ..., f_Q(b_Q^{|Q|}) \right\}$
- If P bids b_P^0 and Q bids $b_Q^1 \Rightarrow$ P wins
- Threshold bid $\forall e$ in P: $T_e \geq \frac{L}{|P|} + \epsilon L$, the total payment is $L(1 + |P|\epsilon)$

Theorem

Any truthful mechanism on a graph that contains either an s-t arc or three node disjoint s-t paths and satisfies the independence, sensitivity and edge and path autonomy properties can be forced to pay $L(1 + k\epsilon)$, where the winning path has k edges and costs L, even if there is some node-disjoint path of cost $L(1 + \epsilon)$

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* Note: Min-Function Mechanisms have **bad** behavior as VCG

Extention by Elkind et al.

- Every truthful mechanism can be forced to overpay just as hardly as VCG in the worst case
- Extend the non-frugality result of previous theorem to all graphs and without assuming the mechanism has the desired properties
- A commonly known probability distribution on edge costs: Bayes-Nash Equilibrium

Theorem

For any L, $e > 0$, *there are bid vectors* b_P, b_Q *such that* $b_P = L$, $b_Q = L + \epsilon$ *and the total payment is at least* $L + \frac{\epsilon}{2}\min(n_1, n_2)$, *where* $n_1 = |P|$ *and* $|Q| = n_2$

Results

- Min-Function Mechanisms have **bad** behavior as VCG
- An exceptional mechanism is truthful mechanism and satisfies the desired properties (edge, path autonomy, independence and sensitivity), but is not min function mechanism

Budget Feasible Mechanisms

Model (Singer 2010)

- There are n agents $a_1, ..., a_n$
- Each agent has a private cost $c_i \in \mathbb{R}_+$ for selling a unique item
- There is a buyer with a budget $B \in \mathbb{R}_+$
- A demand valuation function $V : 2^{[n]} \to \mathbb{R}_+$

▷ A mechanism is **budget feasible** if the payments it makes to agents do not exceed the budget

▷ Goal: to design an incentive compatible budget feasible mechanism which yields the largest value possible to the buyer:

$$\text{maximize V(S)}$$

$$\text{while } \sum_{i \in S} c_i \leq B$$

Budget Feasible Mechanisms

Goals

1. Computation Efficient Mechanism
2. Truthful Mechanism
3. Budget Feasible Mechanism
4. a-approximate Mechanism

Examples:

* Knapsack: find a subset of items S which maximizes $\sum_{i \in S} v_i$ under Budget
* Matching: find a legal matching S which maximizes $\sum_{e \in S} v_e$ under Budget
* Coverage: find a subset S which maximizes $\bigcup_{i \in S} T_i$ under Budget

BFM - Question

? Which **utility functions** have budget feasible mechanisms with reasonable approximation guarantee

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? Which **utility functions** have budget feasible mechanisms with reasonable approximation guarantee

* Result: For any **monotone submodular function** there exists a randomized truthful budget feasible mechanism that has a constant factor approximation
  - This result is developed by proportional share mechanisms

Proportional Share Allocation

Proportional share mechanism: shares the budget among agents proportionally to their contributions.

- Sort: $c_1 \leq c_2 \leq ... c_n$
- Allocate: $c_k \leq \frac{B}{k}$
- Set allocated: $f_M = \{1, 2, ..., k\}$
- For every agent, payment: $\min\left\{\frac{B}{k}, c_{k+1}\right\}$

Then, summing over the payments that support truthfulness satisfies the budget constraint.

Theorem

For $f(S) = |S|$ the mechanism is a 2-approximation

Theorem

For $f(S) = |S|$, no budget feasible mechanism can guarantee an approximation ratio better than 2

General Submodular Functions

- Nondecreasing submodular utility functions (taking computation limitations into account)
- May require exponential data to represented $\Rightarrow$ the buyer has access to a value oracle (given a query $S \subseteq [n]$ returns $V(S)$)
- **Marginal contribution** of agent i: $V_{i|S} := V(S \cup i) - V(S)$
- $V(S) = \sum_{i \leq k} V_i$
- Sort: $\frac{V_1}{c_1} \geq \frac{V_2}{c_2} \geq ... \geq \frac{V_n}{c_n}$
- Allocate: $c_i \leq \frac{B \cdot V_i}{V(S_i)}$
- For every agent, payment: $\min \left\{ \frac{B \cdot V_i}{V(S_i)}, \frac{V_i \cdot c_{k+1}}{V_{k+1}} \right\}$

Charecterizing Threshold Payments

Definition

The marginal contribution of agent i at point j is

$$V_{i(j)} = V(T_{j-1} \cup \{i\}) - V(T_{j-1})$$

where T_j denotes the subset of the first j agents in the marginal contribution-per-cost sorting over the subset $N \setminus \{i\}$

Lemma (Payment Characterization)

The threshold payment for f_M is $\max_{j \in [k+1]} \{\min\{c_{i(j)}, \rho_{i(j)}\}\}$

- $c_j \leq \frac{V'_j \cdot B}{V(T_j)}$
- $c_{i(j)} = \frac{V_{i(j)} \cdot c_j}{V'_j}$ (Agent i appears in the jth position)
- $\rho_{i(j)} = \frac{V_{i(j)} \cdot B}{V(T_{j-1} \cup \{i\})}$ (Agent i is allocated at stage j)

Budget Feasible Mechanisms

Theorem

For any **monotone submodular function** *there exist a* **randomized universally truthful** *budget feasible mechanism with a constant factor approximation ratio. Also, no budget feasible mechanism can do better that* $2 - \epsilon$ *for any fixed* $\epsilon > 0$

- Universally truthful: randomization between truthful mechanisms
- Constant factor $\approx 117,7$

* Knapsack: 5-aproximation budget feasible mechanism

* Matching: $(\frac{5e-1}{e-1})$- aproximation budget feasible mechanism

* Coverage; fails

Budget Feasible Mechanisms - Open Questions

? **Constant factor** approximation for **subadditive functions** using demand queries

? **Other classes of functions** have budget feasible mechanisms

? Budget feasible mechanisms that are not based on **proportional share mechanisms**

Learning on a Budget: Posted Price Mechanisms

- **Online** procurement markets
- Mechanism makes agents "take-it-or-leave-it" offers
- Agents are drawn sequentially from an **unknown distribution** (describes the costs)
- For agent i the mechanism posts a price p_i
- If $p_i \geq c_i \Rightarrow$ agent accepts & buyer receives the item
- Technical Challenge: to learn enough about distribution under the budget
    - High offers $\Rightarrow$ exhaust Budget
    - Low offers $\Rightarrow$ exhaust Pool of Agents

Learning on a Budget: Posted Price Mechanisms

Model (BKS 2012)

- There are n agents $a_1, ..., a_n$
- Each agent has a private cost $c_i \in \mathbb{R}_+$ for selling a unique item
- There is a buyer with a budget $B \in \mathbb{R}_+$
- A demand valuation function $V : 2^{[n]} \to \mathbb{R}_+$
- Online arrival of agents
- Exist n different time steps: in each step $i \in [n]$ a single agent appears
- Mechanism makes a decision: based on the information it has about the agent & the history of the previous $i - 1$ stages
- How the order of agents is determined?
  1. Adversarial model
  2. Secretary model
  3. i.i.d model

Learning on a Budget: Posted Price Mechanisms

Theorem

For any nondecreasing submodular procurement market there is a randomized posted price budget feasible mechanism which is universally truthful and is $O(\log n)$-**competitive**

Idea

- Choose $\tau \in [0, n]$ agents
- Finds the agent with the highest value: $v' = \max_{\{a_i : i \leq \tau\}} f(a_i)$
- Estimate: $t = g(v')$
- For each $a \in N \setminus \{a_1, ..., a_\tau\}$
  - Offer the agent $p = \frac{B}{t} \cdot (f(S \cup \{a\}) - f(S))$
  - If a accepts, add it to S & set $B' = B' - p$

* Combine with Dynkin's algorithm (secretary problem)

More Results

Theorem

For the case of $f(S) = |S|$. The utility function f is a symmetric submodular function. The algorithm is **constant***-competitive when agents are identically distributed. In fact, with probability at least 1/2, the number of offers accepted is at least $c \cdot (B/p_l)$*

Theorem

In the **bidding model***, for any nondecreasing submodular utility function there is a universally truthful budget feasible mechanism which is* $O(1)$-**competitive**

Learning on a Budget: Posted Price Mechanisms - Open Question

? There exists a $O(1)$-competitive posted price mechanism in the nonsymmetric submodular case

References

G. Christodoulou and E. Koutsoupias, "Mechanism Design for Scheduling", *Bulletin of the EATCS*, Vol. 97, pp. 40-59, 2009.

D. C. Parkes, "Online Mechanisms", *Algorithmic Game Theory*,2007.

Y. Singer, Budget Feasible Mechanisms, *Foundations of Computer Science (FOCS), 2010 51st Annual IEEE Symposium on*, pp. 765-774, 2010.

Y. Singer, Budget Feasible Mechanism Design, *ACM SIGecom Exchanges.*, vol. 12, no. 2, pp. 2431, 2014.

A. Badanidiyuru, R. Kleinberg and Y. Singer, "Learning on a Budget: Posted Price Mechanisms for Online Procurement", *Proceedings of the 13th ACM Conference on Electronic Commerce*, pp. 128-145, 2012.

References

A. Archer and É. Tardos, "Frugal Path Mechanisms", *Proceedings of the 13th Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 991-999, 2002.

E. Elkind, A. Sahai and K. Steiglitz, "Frugal in Path Auctions", *Proceedings of the Fifteenth Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 701-700, 2004.


THANK YOU