# An Optimization Model for the Strategic Design of a Bicycle Sharing System: A Case Study in the City of Athens

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## ABSTRACT

Bicycle sharing systems comprise fleets of bicycles made available for shared use at low cost. Bicycles are distributed in stations spread within an urban area and may be picked up and dropped off at any station under flexible short-term rental schemes. The design of bicycle sharing systems raises several optimization problems as it involves the determination of the number, capacity and location of bicycle station facilities as well as the bicycle fleet size along with the allocation of bicycles among stations. These design decisions are subject to several variables, restrictions and dependencies, such as the predicted user demand patterns, the synergies among the bicycle sharing system and the public transportation network, the budget available for setting up the system, etc. In this paper we investigate the problem of the strategic design of bicycle sharing systems. We develop and solve a mathematical model for determining the location and number of vehicle stations, the optimal fleet size and the distribution of bicycles to the stations of a bicycle sharing system, taking into account the user demands and the investment cost. In order to test and validate our approach in realistic settings, we applied the proposed model in the design of a bicycle sharing system in the city of Athens, Greece.

## Keywords

bicycle sharing system, strategic design, Athens, optimization



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## 1. INTRODUCTION

Bicycle Sharing Systems (BSSs) are networks of public use bicycles distributed around a city for use at low cost. The BSSs comprise short-term urban bicycle-rental schemes that enable bicycles to be picked up at any bicycle station and returned to any other bicycle station. Current BSSs deploy bicycles which may be picked up and returned at specific locations (docking stations) and typically employ some sort of customer authentication/tracking usually through the use of an electronic subscriber card [9]. Recent research analyzed the factors affecting the success of BSSs. Those involve bicycle station location, cycling network infrastructure (bike paths) and the operation of a bicycle redistribution system.

Vogel et al. [10] identify three main issues related to the design, management and operation of bicycle as well as car sharing systems. The proposed design and management measures aim at alleviating imbalances in the availability of bicycles/cars and are distinguished into three separate planning horizons: (i) Strategic network design comprising decisions about the location and the number of stations as well as the bicycle/car stock at each station. (ii) Tactical incentives for customer-based distribution of bicycles/cars i.e., incentives given to users so as to leave their bicycle/car to a station different to that originally intended and (iii) Operational repositioning of bicycles/cars based on the current state of the stations as well as aggregate statistics of the stations' usage patterns.

Given the complexity of bicycle facility planning and the importance of station distribution for operating BSSs, formal approaches are needed to model the problem variables and derive optimal solutions with respect to minimizing investment cost and maximizing utility for the users. Among others, optimal solutions should determine the number, location and capacity of the stations and, optinally, the setup of bicycle lanes. On the other hand, equally important for BSSs success is to guarantee bicycle availability. Each station must carry enough bicycles to increase the possibility that each user can find a bicycle when needed. Therefore, measures of service quality in the BSS include both the availability rate (i.e., the proportion of pick-up requests at a station that are met by the bicycle stock on hand) and the coverage level (i.e., the fraction of total demand at both origins and destinations that is within some specified time or distance from the nearest station).

Several approaches have been proposed in the literature to solve the main issues arising in bicycle/car sharing systems (for a survey see [3], [4]). In this paper we investigate the problem of strategic design of bicycle sharing systems. We develop and solve a mathematical model for determining the location and number of bicycle stations, the optimal fleet size and the distribution of bicycles to the stations of a BSS taking into account the user demands and the cost of the investment. The approach estimates the demand by analyzing data which is easy to obtain for any concerned area of interest, therefore it is generic and easy to fine-tune. Furthermore, we apply the proposed model for the design of a bicycle sharing system in the center of Athens. The rest of this paper is organised as follows. Section 2 presents the related work, Section 3 discusses the formulation of the problem and the solution approach while Section 4 presents the results from the application of the model for the design of a bicycle sharing system in the city of Athens.

## 2. RELATED WORK

The term "strategic design" is directly linked to the classic *Facility Location* problem: the optimization of opening facilities within a region/city, in order to satisfy the desired objective. This can be the minimization of the overall facility build cost, the minimization of transportation cost or maximum distance to the facilities, or the demand coverage. However, as Vehicle Sharing Systems concern usage of the facilities (stations) which fluctuates within the day, correlates the departure/pick-up to the arrival/drop-off station and also depend on existing infrastructure (public transportation), the formulation often becomes more complex.

Lin and Yang ([5]) have been the first to investigate the problem of strategic design of bicycle sharing systems. The problem investigated is the following: given a set of origins, destinations, candidate sites of bike stations and the stochastic travel demands from origin to destination, the problem's output comprises the location of bike stations, the bicycle lanes needed to be setup and the paths to be used by users from each origin to each destination, the objective being to minimize the overall system cost. The problem has been formulated as an integer nonlinear program and was solved by CPLEX on a small instance. Recognizing the complexity of the bicycle sharing system design optimization which precludes exact solutions for instances of realistic size, Lin et al. ([6]) approached the system's design as a hub location inventory problem that takes the coverage level into consideration and proposed a greedy algorithm for efficiently solving it. The overall solutions cost is calculated utilizing the mathematical cost model introduced in an earlier study [5]. When testing the algorithm in test instances for which enumeration is possible, the heuristic solution has been found within a 2% gap from the optimal. Correia and Antunes ([2]) addressed the optimization problem of selecting sites for locating depots in order to maximize the profits of a one-way car sharing system. Revenues are generated from renting the vehicles against some price rate while several types of expenses are considered (maintenance costs, vehicle depreciation costs and vehicle relocation costs). Three mixed integer programs (MIP) have been modeled which determine the optimal number, location, and capacity for the depots.The optimization models have been tested in a case study involving the municipality of Lisbon, Portugal.

Nair and Miller-Hooks ([8]) consider the problem of determining the best locations for the stations, as well as the capacity and initial number of vehicles of each station subject to a budget constraint. The solution's quality is given by the expected revenue, seen as a linear function of user flows. The authors propose an equilibrium network design model to formulate the problem of determining an optimal configuration of the BSS. The model takes the form of a bi-level, mixed-integer program i.e. a program where the values of some variables are optimal solutions of another optimization problem which is called the lower-level problem.

Boyaci et al. ([1]) proposed a generic model for supporting the strategic (number and location of required stations) and tactical (optimum fleet size) decisions of one-way car-sharing systems by taking into account operational decisions (i.e. relocation of vehicles). The authors formulated a mathematical model and conducted sensitivity analysis for different parameters. The objective function seeks to maximize the overall profit which considers the revenue generated from vehicle rentals in addition to user costs and system costs The proposed model has been applied for planning and operating a station-based EV-sharing system in the city of Nice, France. Martinez et al. ([7]) formulated a mixed integer linear program (MILP) aiming to optimize the location of bicycle stations and the fleet dimension. This study also considered bike relocation operations among docking stations.

## 3. A CAPACITATED FACILITY LOCATION APPROACH

The Capacitated Facility Location is a demand covering via facility opening problem. The input is a weighted set of demand points and a set of potential facility opening points, all laying on a metric space, and the goal is to determine the optimal number, positions and capacities of facilities that can satisfy the total of the demand, given the cost of opening and expanding each facility ([11]).

Our work concentrates in taking advantage of the relative simplicity of this well-established problem and measure the quality of solutions which are obtained when adopting this static approach. It is, therefore, necessary to preprocess the fluctuating demand within our region of interest as to obtain the correct weight of the demand points. Following the work of Wu et al. ([11]), we formulate the mixed-integer Linear Program; we will only alter the objective function and insert an additional constraint to fit our needs.

### 3.1 The Mixed Integer Linear Program formulation

Let us first describe the input variables, as used for the Program formulation.

- $I$: the set of demand points
- $\mathcal{W}(I)$: the weight of the demand points inserted in the Linear Program. The exact selections of $\mathcal{W}$ are extensively explained in the next Paragraph.
- $J$: the set of candidate station locations
- $d_{ij}$: the induced distances of each pair $(i, j) \in I \times J$.

- $f_j$, $\alpha_j$: the facility opening and expansion per capacity unit cost respectively.
- $C_j$, $L_j$: the maximum and minimum allowable capacity for each candidate station location
- $B$: the total available budget

For each pair $(i, j) \in I \times J$, we define $y_{ij}$ as the fraction of the demand of point $i \in I$ satisfied by facility $j \in J$. Eventually, we desire that $\sum_{j \in J} y_{ij} = 1$ (1), in other words point $i$'s demand should be covered by some of the stations. Of course, in order for some station $j$ to cover demand, it should be opened and have a capacity $c_j$ which relates to each demand point $i$ through the fraction $y_{ij}$ and it should be analogous to the weight $\mathcal{W}(i)$; thus, we get constraint (2). Constraints (3) and (4) concern the opening or not of each potential station and (5) simply ensures the natural meaning of quantities $y_{ij}$. The final is the budget constraint (6). Overall, we have:

$$\text{maximize:} \quad \sum_{i \in I} \sum_{j \in J} \frac{\mathcal{W}_i}{d_{ij}} y_{ij}$$

$$\text{s.t.:} \quad \sum_{j \in J} y_{ij} = 1, \quad i \in I \tag{1}$$

$$\sum_{i \in I} \mathcal{W}_i y_{ij} \le c_j, \quad j \in J \tag{2}$$

$$L_j x_j \le c_j \le C_j x_j, \quad j \in J \tag{3}$$

$$x_j \in \{0, 1\}, \quad j \in J \tag{4}$$

$$y_{ij} \ge 0, \quad i \in I, j \in J \tag{5}$$

$$\sum_{j \in J} f_j x_j + \sum_{j \in J} \alpha_j c_j \le B \tag{6}$$

#### 3.1.1 The objective function

The MILP presented in [11] comprises a different objective function:

$$\text{minimize:} \quad \sum_{i \in I} \sum_{j \in J} d_{ij} y_{ij} + \sum_{j \in J} f_j x_j + \sum_{j \in J} \alpha_j c_j.$$

However, there is the problem of balancing the effect of the budget terms, which correspond to money, with the leading coverage term, which relates to the metric space. Intuitively, we inserted the budget constraint, which absolutely captures reality. As a second step, and due to the minimization of the remaining $\sum_{i \in I} \sum_{j \in J} d_{ij} y_{ij}$ is trivial and of no importance, we inverted the distances and favoured (once again in a natural way) the usability and potential expected revenue of high demand points satisfied by closely positioned, high capacity stations.

#### 3.1.2 Selecting and normalizing $\mathcal{W}$

The weight-capacity constraint (2) gives the relation between the capacity of an opened station and the demand it should cover. Having in mind that $y_{ij}$ are normalized, $\mathcal{W}$ should be capacity-normalized, as to properly induce the required $c_j$. Therefore, if $w_i$ is the actual demand of point $i$, then $\mathcal{W}_i = \frac{\max\{C_j\}}{\max\{w_i\}} w_i$. In other words, we force that the maximum weighted demand point $i^*$ covered only by the maximum capacity station $j^*$ induces this maximum capacity $C_j$ for $j^*$. This works best if all maximum capacities are equal; this is not counter-intuitive, so in the case studies we have $C_j = C, \forall j \in J$.

#### 3.1.3 The budget minimization MILP

We should note that by simply declaring the budget constraint 6 as the objective function, maintaining the set of constraints (1)–(5), we obtain the minimum-budget solution which satisfies all demand. This MILP variant can be used to then solve our main MILP for a minimum-budget, maximum-objective solution (bi-level formulation).

### 3.2 Real Data processing and System Setup

In order to present valuable and rigid results, we chose to collect real *General Transit Feed Specification* (GTFS) data courtesy of the Athens Urban Transport Organisation. These involve real time passages of metro, thermal and trolley buses and tram through over 7,500 stops, for each day of the week. These frequencies correspond (ideally) to the demand for transportation, therefore we directly extract the input set $I$ and the weights $\mathcal{W}$ from this data. Let use note that in all cases, we took into consideration the difference in capacity between buses, metro and tram; therefore the relative demand weights are *bus-1, metro-5 and tram-2*.

For all purposes, we extracted the *hourly arrival rates* for every stop and every day of the week; all analysis was chosen to be done for Friday, when the week's maximum overall rates are spotted. The 21 actual timeslots defined were *early morning (<6:00), hourly intervals for 6:00. to 24:00 (18 slots), 24:00-2:00, and late night (>2:00)*. As a quite extensive process of pre-processing was needed, and the capability of moving back and forth from selecting points on google maps (to create a realistic system proposition) all the way to getting the results as a vector of station capacities, we selected MATLAB optimization package as our LP solver. We proceeded as follows:

1. We based our study on Athens' Traffic Ring. The Traffic Ring surrounds the city centre and marks an area where -due to the high demand to reach- it is not allowed for all private cars to enter at all times (even/odd day-car licence rule).
2. We confined to a small extension of Athens' Traffic Ring. The extension for our pursposes was based on two remarks:
   (a) There is no reason to have a high demand point on the boundary of the under-study area; such points probably involve a metro station, ideal to attract demand from the outside of the centre.
   (b) Special facilities a little outside the Traffic Ring (mostly University facilities and some student houses) also "pull" the boundary towards them.
3. 688 stops-demand points were taken into account and were clustered to 300, by setting a threshold a little less than $50m$ as the closest two demand points may lie. For each cluster, the demand was accumulated from all points of the cluster.
4. 272 potential station locations were manually selected. Points Of Interest and neighbourhood squares were the first to include as such, as well as a realistic set of spots to maintain a somewhat more even distribution of potential locations.
5. The facility opening/expansion cost ratio is set to 5:1 for this case. It is a rough estimation based on related work and published figures. Along with presenting our

results, we will explain what to anticipate if the ratio is changed.

6. Finally, as an attempt to be realistic, we confined each station to feature at least 10 and at most 50 docks.

### *3.2.1 Legend – figure guide*

In what follows, we will present illustrations of the proposed solutions drawn from solving the Linear Program.

- The demand points are presented as empty blue-grey circles.
- The larger an empty circle is, the greater the demand at the associated point.
- The potential stations are shown as red filled circles.
- The larger a red circle is, the greater the capacity suggested by the solver for the associated point.

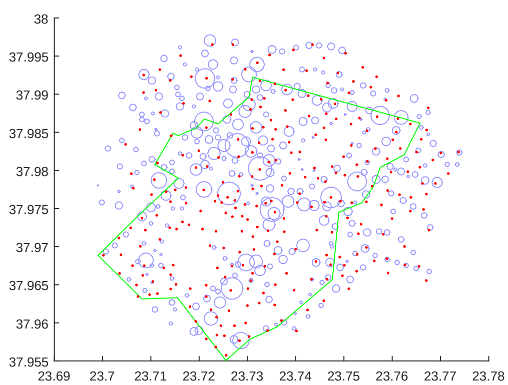



Figure 1: Athens' Ring (green boundary), demand points and candidate stations.

## 4. RESULTS AND DISCUSSION

In this Section, we will present two main results: a justification for statically approaching the strategic design problem, without significant loss of information due to the demands fluctuation; and a measure for monitoring the quality of a series of solutions with increasing budget.

### 4.1 Robustness under static demand conversion

Having a total of 21 (or any other selected number of) timeslots to give the picture of one point's demand throughout a day, it is reasonable to presume that fully satisfying the whole day's demand means satisfying each timeslot's demand. Consequently, we may consider a specific timeslot, solve the capacitated facility location problem for this interval, and synthesize the 21 solutions to get the unique final solution. This procedure is costly, even if the final synthesis execution time is negligible. In order to diminish this running time, a natural question is whether one run of the MILP is good enough to avoid another 20 runs.

We selected 3 different synthesis functions, in fact, statistic measures and compared the results as follows, $\boldsymbol{\mathcal{W}}_t$ being the demand vector of timeslot $t$:

- Mean value: $\texttt{SOL}(\overline{\boldsymbol{\mathcal{W}}_t})$ *vs.* $\overline{\texttt{SOL}(\boldsymbol{\mathcal{W}}_t)}$;
- Maximum value: $\texttt{SOL}(\max\{\boldsymbol{\mathcal{W}}_t\})$ *vs.* $\max\{\texttt{SOL}(\boldsymbol{\mathcal{W}}_t)\}$;
- "Mixed" value, the minimum of the maximum value and the mean value shifted by the standard deviation, $\min\{\max, \text{mean} + \text{sigma}\}$: $\texttt{SOL}(\text{mxd}\{\boldsymbol{\mathcal{W}}_t\})$ *vs.* $\text{mxd}\{\texttt{SOL}(\boldsymbol{\mathcal{W}}_t)\}$.

In other words, we study the effect of altering the stage of synthesis. It is expected that the mean of the solutions $\overline{\texttt{SOL}(\boldsymbol{\mathcal{W}}_t)}$ is a feasible point of the mean-demand MILP (we may divide constraint 2 by the number of timeslots chosen), however, due to the discretization of the opening station variables $x_j$, the linearity is impaired.

Since the number of stations is large, as the comparison result we consider the histograms of the occurring differences between the suggested (synthesized) capacities. The experiment was done for different budgets (between 2000-3000), and three different metrics: squared and normal euclidean distances, as well as a 50%-50% mixed euclidean-Manhattan distance[1].

Figure 2 illustrates some of the results, though all experiments lead to the same conclusion: under any of the 3 synthesis functions, the solution obtained when applying the synthesis function to the demands, before solving a single MILP is essentially the same as synthesizing the solutions of 21 run MILPs.

Moreover, the differences are reasonable: due to the minimum capacity constraint from one hand -a station opened with 10 docks for, say 15 of the 21 timeslots, and closed for the remaining 6, will yield an average 7.14 docks, a quantity which cannot be obtained when solving the LP. Analogously for the maximums, among the 21 solutions different stations are chosen to be opened, so the maximum capacity for more than the ideal-number-to-open stations is greater or equal than 10. This explains the significant amount of stations differentiated by 10 docks, however, excluding this inevitable phenomenon, the rest of the stations present no significant differences in the proposed capacities. Finally, selecting the mixed measure, the result resembles the result for the maximum, with a favourable shift and smoothing of the larger differences. Thus, *we shall calculate all solutions based on the day's "mixed" demand for each point.*

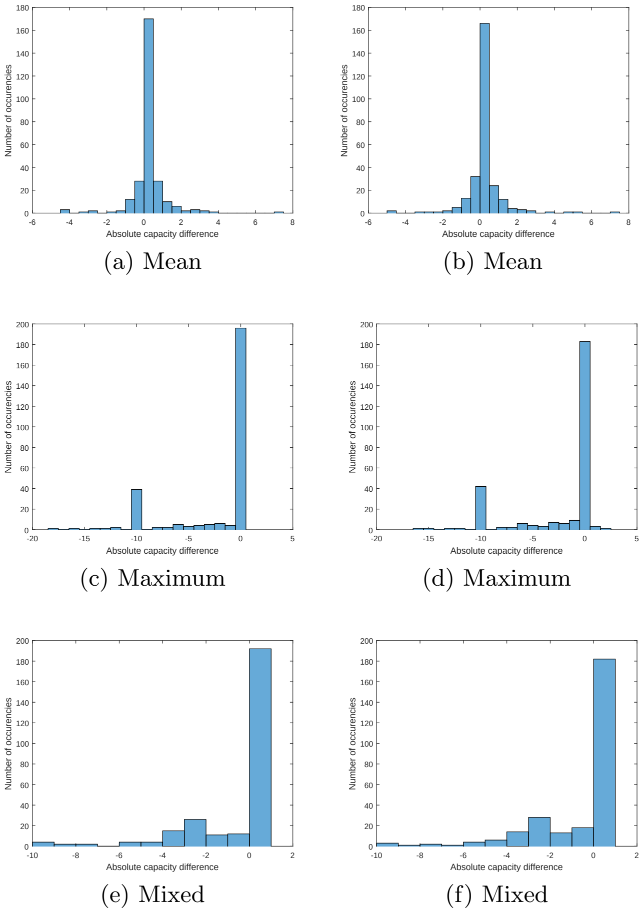
#### *4.1.1 The effect of a cut-off distance*

As previously, let the points of interest (demand, potential station) define a metric space. In such systems, an operational fact for users, who invoke demand at $i$, is the existence of a *cut-off distance*, beyond which all stations $j$ are considered *not* to satisfy their demand. This can be inserted in our MILP's objective function as defining $\mathcal{W}_i/d_{ij} = 0$ whenever $d_{ij} > d_{cut}$, plus adding an extra constraint for the upper bounds of $y_{ij}$: $d_{ij} > d_{cut} \implies y_{ij} \leq 0$.

Once again, we compared solutions for different budgets, the three distances mentioned earlier (euclidean, squared euclidean and "50-50 mixed" euclidean-Manhattan) and different cut-off distances, ranging from $1km$ down to the *threshold* distance, the cut-off distance below which the MILP is infeasible. Our conclusions are the following:

1. When using the squared euclidean distance, the tail of the terms $\frac{1}{d_{ij}}$ is diminished comparing to non-squared distances; it introduces practically a cut-off filter, so no $d_{cut}$ significantly alters any result.

[1] $d^M(x,y) = \frac{||x-y||_1}{2} + \frac{\sqrt{||x-y||_2}}{2}$.




(a) Mean (b) Mean

(c) Maximum (d) Maximum

(e) Mixed (f) Mixed

Figure 2: Budget: 2500. Distance: squared euclidean (left column), mixed (right column).

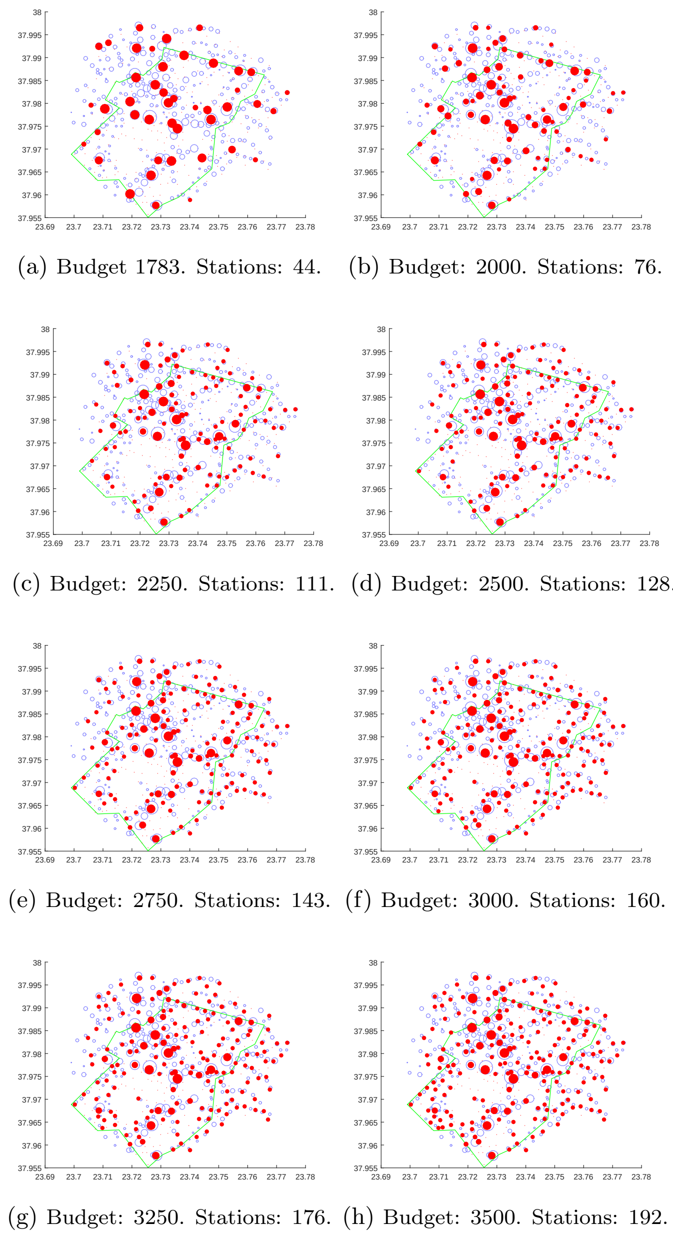
2. The differences in the solutions were not so important as to indicate a "better" selection for the metrics. Also, there is an inherent to the point sets $I$ and $J$ low threshold of $685m$ for $d_{cut}$, below which the LP becomes infeasible, i.e. the demand constraints cannot be satisfied, no matter how big the budget is.

The above still left us free to settle to what better describes the point-to-point walking distances in Athens, plus adopt the following threshold as cut-off distance: *we shall calculate all solutions based on the 50%-50% "mixed" euclidean-Manhattan distance for our metric space, cut-off at 0.7km.*

## 4.2 Solution behaviour vs. budget selection

Having established an initially good quality of our solutions, we proceeded to determine at is the minimum budget for which the demand is covered. Using the min-budget variant of the MILP, we obtained **MinBudget=**1783. We then proceeded to solve the LP with a first budget constraint of this very quantity and gradually increasing (steps of 250) to get a series of images for the proposed Bike Sharing System (Figure 3 – the actual Athens' Ring is marked in green).

The ending of the series at the budget of 3500 is not accidental. For every budget greater or equal to 3499, the solution remains the same; we may say there is a *budget saturation* which is due to the globally optimal partitioning of quantities $y_{ij}$ for each $i \in I$ and with respect to the minimization of the objective function: greater budget admits greater station capacities, which actually relaxes the weight-capacity constraint and therefore the convex combinations of the demand coverage are displaced towards one of their marginal points ($y_{ij^*} = 1$ for some $j^*$ and $y_{ij} = 0$ for $j \in J \setminus \{j^*\}$), as those are favoured by the multiplication in objective function. Once the appropriate marginal points all become feasible, the objective function meets its global minimum.



(a) Budget 1783. Stations: 44. (b) Budget: 2000. Stations: 76.

(c) Budget: 2250. Stations: 111. (d) Budget: 2500. Stations: 128.

(e) Budget: 2750. Stations: 143. (f) Budget: 3000. Stations: 160.

(g) Budget: 3250. Stations: 176. (h) Budget: 3500. Stations: 192.

Figure 3: Athens' Ring proposed BSS.

## 4.3 Solution quality

It is obvious, even without the figures presented, that with a sufficiently high budget, one achieves the best outcome. We attempt to give a tangible result as to when a non-optimal (or non-saturated) budget can be considered as “good enough”, so to address the more realistic case where limited budget is available.

Suppose that given a budget $B$ the solution (amongst the others) suggests to open Station $j$ with a capacity of $S_j$. If all solutions subject to a relaxed budget constraint (i.e. increased budget) $B' > B$ still suggest the opening of station $j$ with $S'_j \geq S_j$, we may reasonably admit that the opening (at budget $B$) is correct/appropriate/well established.

Using this as an indicator of solution quality, we define the *unfavourable difference* of a solution/capacity vector $\boldsymbol{S}(B_1)$ from $\boldsymbol{S}(B_2)$, $B_1 < B_2$: let $J$ be partitioned into the mutally disjoint sets $J^+_{B_1,B_2}$, $J^0_{B_1,B_2}$ and $J^-_{B_1,B_2}$. It is $j \in J^+_{B_1,B_2} \iff \boldsymbol{S}_j(B_2) - \boldsymbol{S}_j(B_1) > 0$ and analogously for $J^0$ and $J^-$. The *unfavourable difference* is defined as:

$$\mathcal{U}_{B_1,B_2} = -\sum_{j \in J^-} \big(\boldsymbol{S}_j(B_2) - \boldsymbol{S}_j(B_1)\big).$$

In other words, $\mathcal{U} = 0$ means that increasing the budget did not decrease the suggested capacity of any of the stations.

Figure 4 can then be understood to indicate that the difference $\mathcal{U}_{2250,2500}$ between solution for budget 2500 ($\boldsymbol{S}(2500)$) and the previously examined ($\boldsymbol{S}(2250)$) is an order of magnitude less than $\mathcal{U}_{2000,2250}$. The same stands for larger budgets, while the first two comparisons yield bad results. In all, the solution becomes quite steady/good at around the 2250 budget mark.

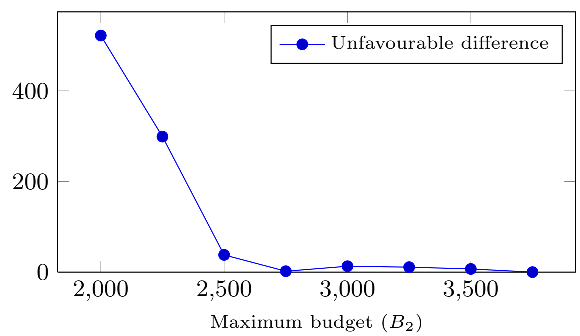



Figure 4: Evaluating the solution quality – BSS.

#### 4.3.1 Different facility opening/expanding cost ratios

The selected ratio 5:1 for the facility opening cost over the facility expansion cost does only quantitatively affect the results. Since the selected objective function does not depend on the facility opening cost, any solution for the 5:1 ratio, say for budget equal to 3000, can be converted to the equivalent solution for a shrunk budget: 160 opened stations consume 800 out of the 3000 of the budget. If the ratio was 2:1, then those 160 stations would require 480 budget units less, so this exact solution is not only a feasible solution for budget 2520 with the 2:1 ratio, it is also the best one.

## 5. CONCLUSION

We examine the adequacy of the Capacitated Facility Location Problem as a model for the strategic design of Vehicle Sharing Systems. Using real GTFS data for the city of Athens and creating a realistic set of potential bicycle stations around the city centre, we used a fixed MILP to obtain marginally feasible solutions w.r.t. a first parameter (budget) and evaluated the solutions’ behaviour when changing the parameter, while fed with a our calculated and clustered demand vector. We finally assess the quality of our solutions based on a concrete measure of consecutive solutions’ similarity. The results indicate a good selection of the particular bi-level formulation and, for our case study, establish an original proposal for the city of Athens.

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